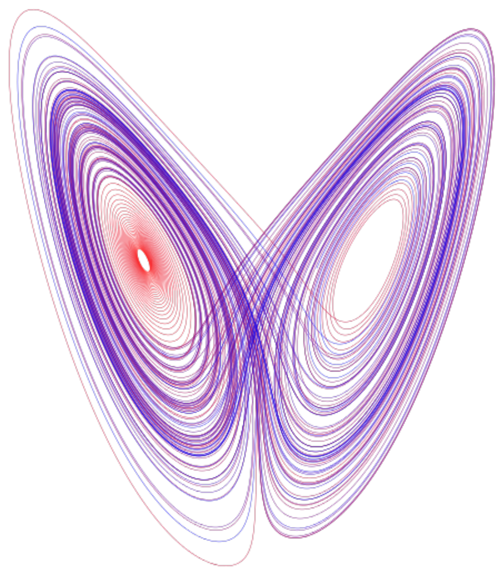
Predictability of Geophysical Flows

Corentin Herbert

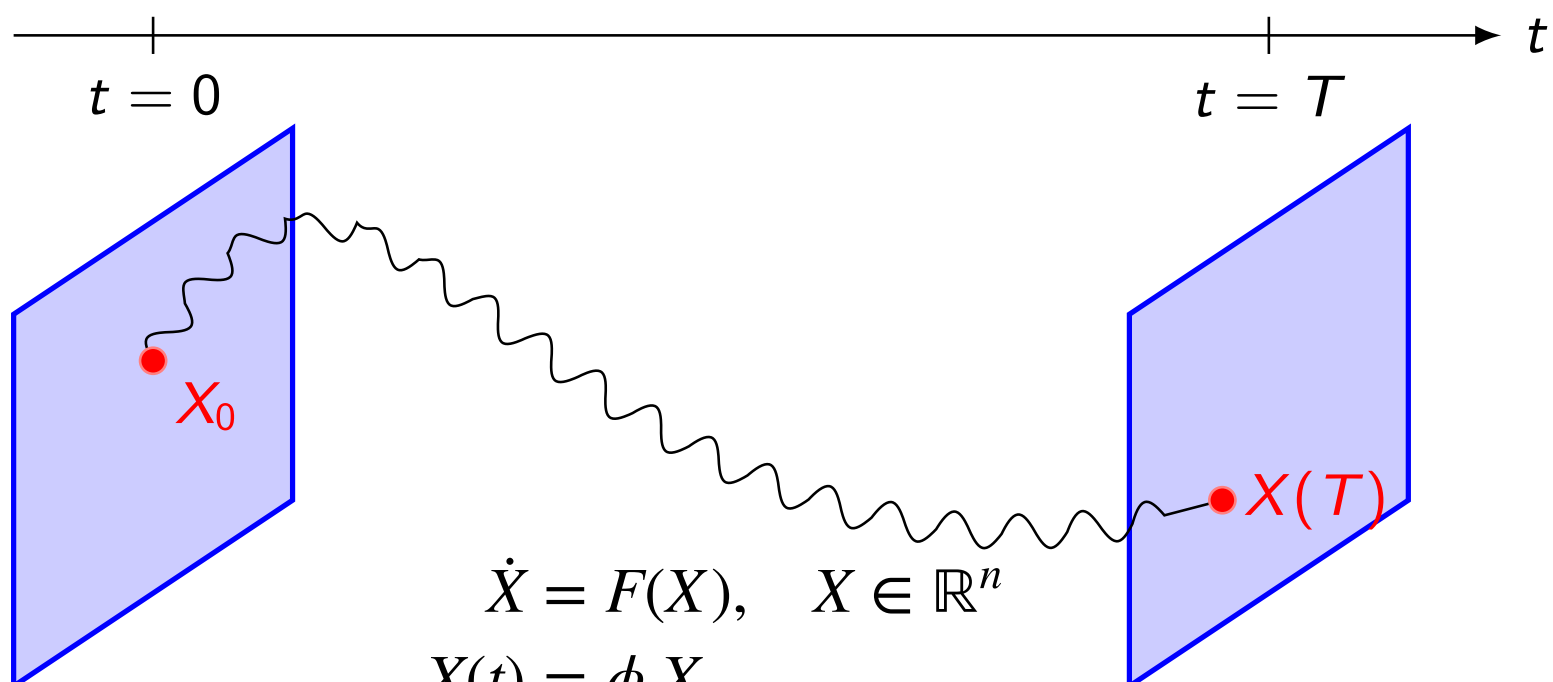


M2 ENS Lyon GFD Week 01/23

Introduction

The prediction problem

Given the laws governing the evolution of a system and knowledge of the present (initial condition), can we predict the evolution of the system in the future?



$$\dot{X} = F(X), \quad X \in \mathbb{R}^n$$

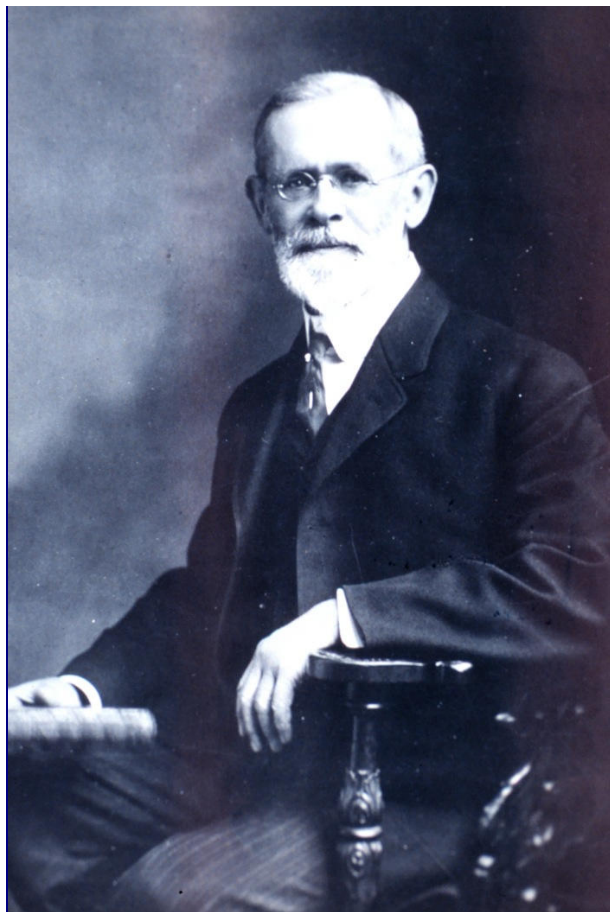
$$X(t) = \phi_t X_0$$

The equations governing the behavior of geophysical flows (atmosphere, ocean) can be written in this form after discretization, with n very large.

A natural example of such a problem for the atmosphere is weather forecasting.

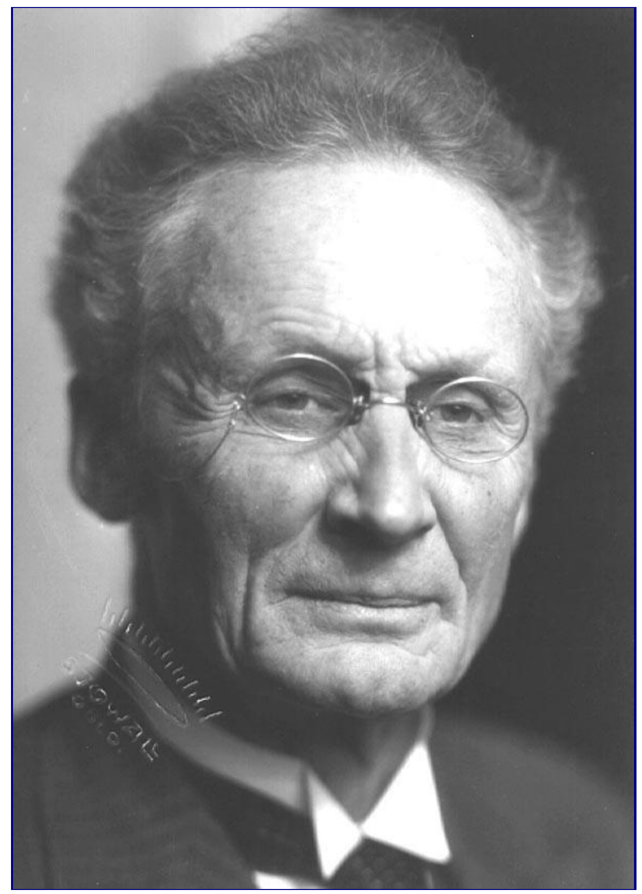
The Founding Fathers of Modern Meteorology

Cleveland Abbe



"Meteorology is essentially the application of hydrodynamics and thermodynamics to the atmosphere." (1890)

Vilhelm Bjerknes



Necessary and sufficient conditions for the solution of the forecasting problem (1904):

1. Knowledge of the initial state
2. Knowledge of the physical laws

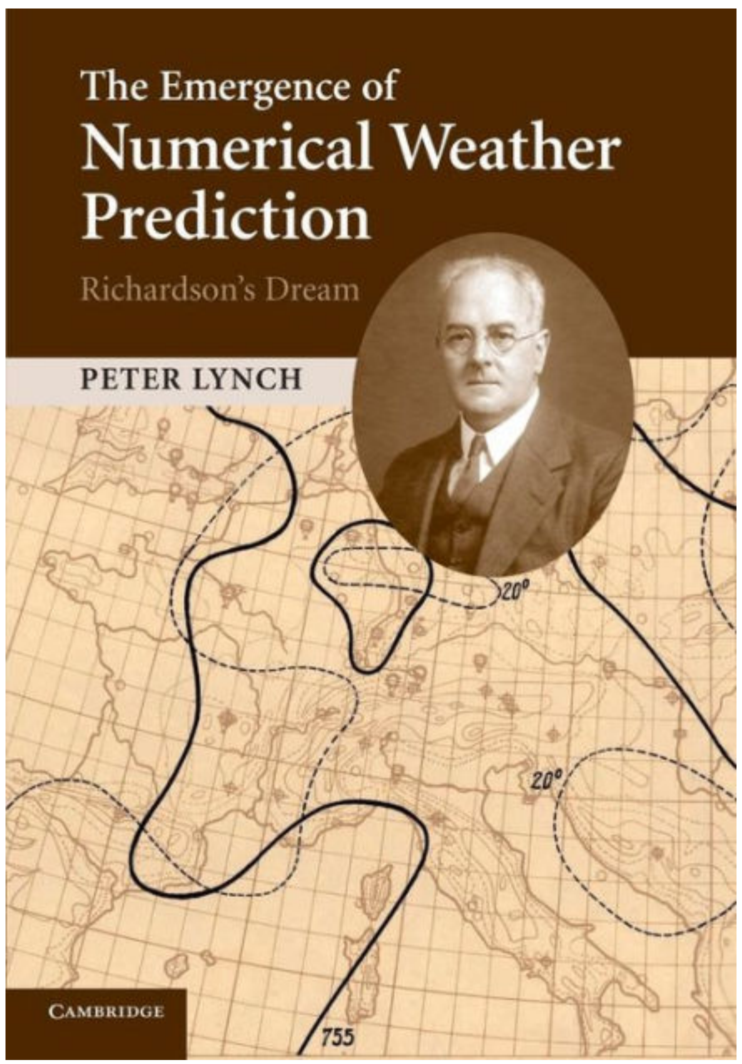
Lewis Fry Richardson

First forecast during WWI:

- By hand!
- 2 years!
- ΔP = 145 hPa in 6 hours!

Did not filter fast oscillations! (gravity waves)

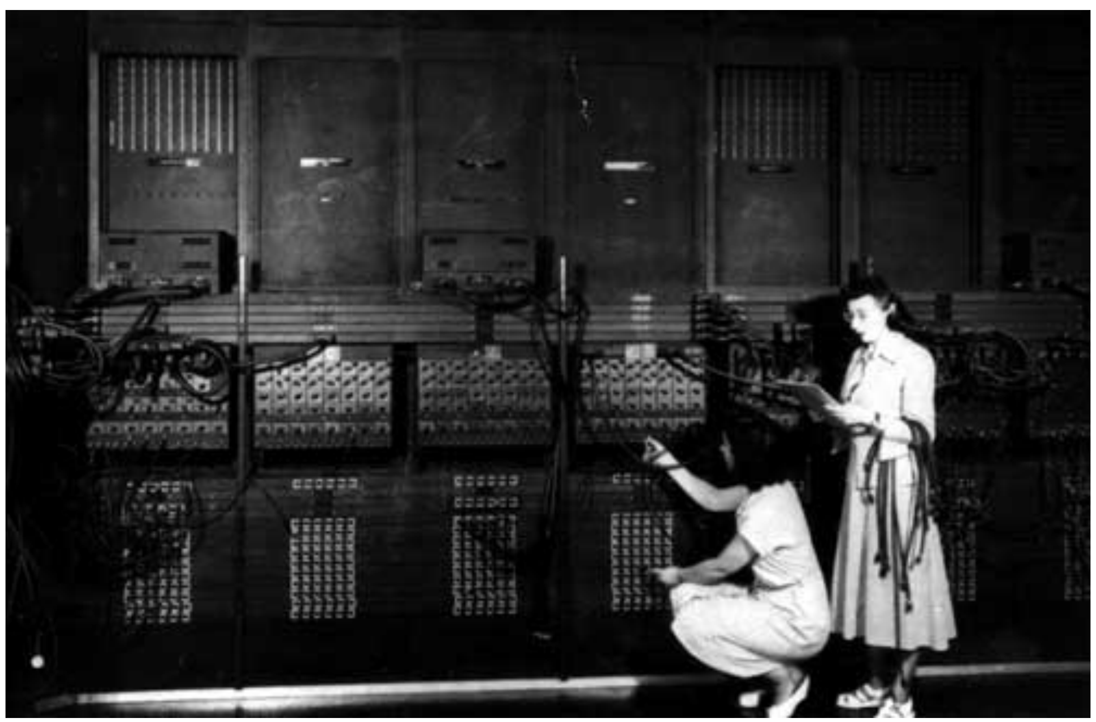
The birth of Numerical Weather Prediction



Richardson's forecast factory: 64000 human computers

The First (Successful) Weather Forecast

The ENIAC machine



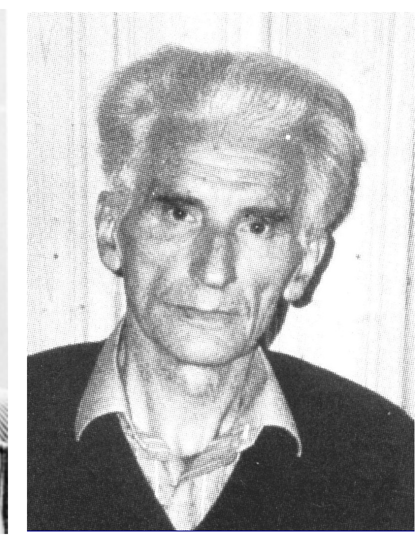
Electronic Numerical Integrator And Computer

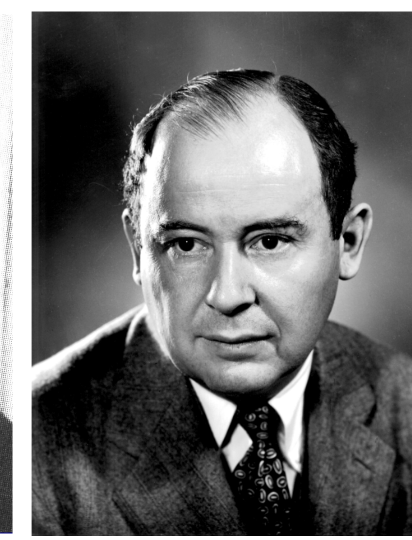
First multi-purpose programmable electronic digital computer

Numerical Integration of the Barotropic Vorticity Equation

By J. G. CHARNEY, R. FJÖRTOFT[1], J. von NEUMANN
The Institute for Advanced Study, Princeton, New Jersey[2]

(Manuscript received 1 November 1950)

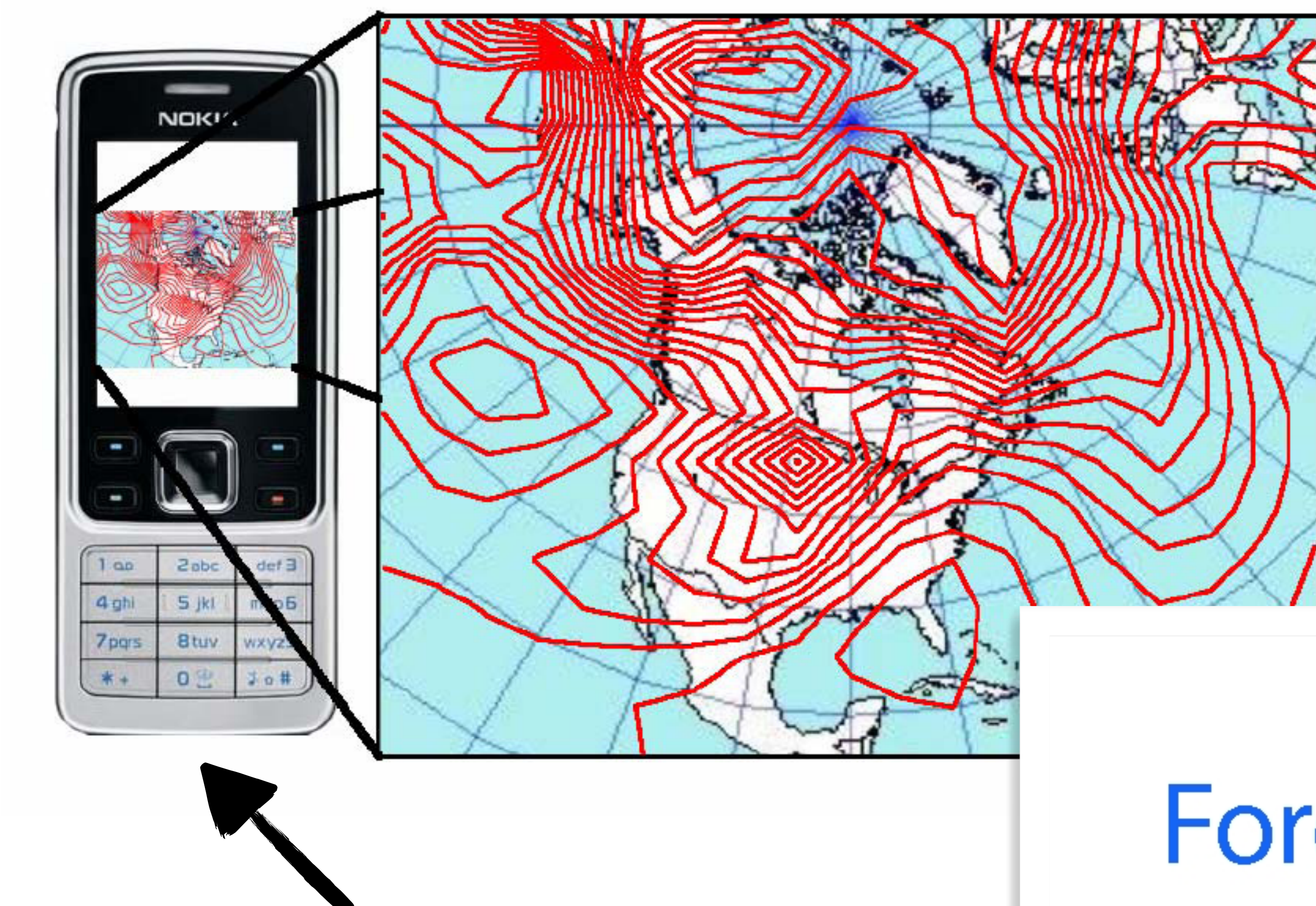




- **Single layer of fluid**
- **Conservation of potential vorticity**

$$\frac{d\zeta}{dt} = 0, \quad \zeta = \nabla \times \mathbf{v} + f$$

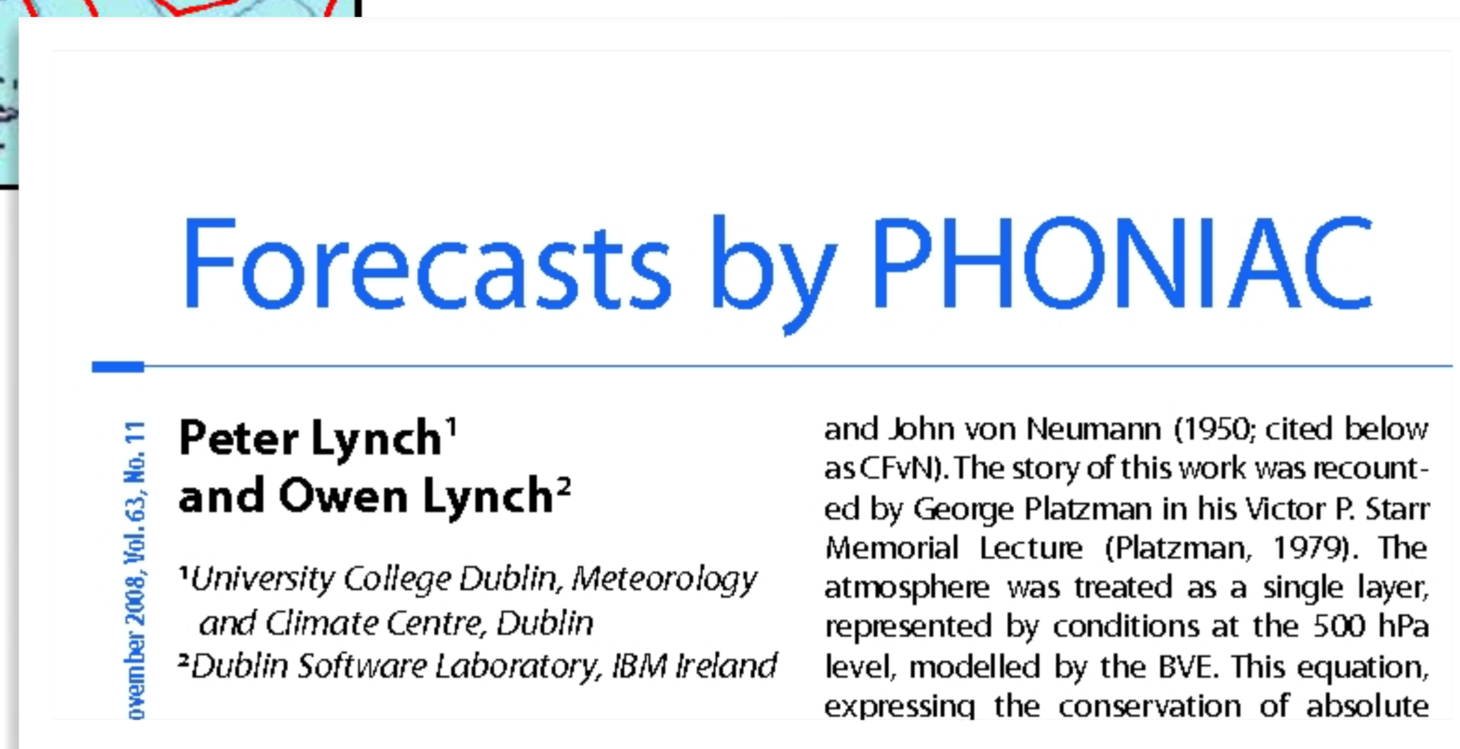
Miniaturization….



237 MFLOPS

ENIAC: ~5 kFLOPS

Weather, 2008



Forecasts by PHONIAC

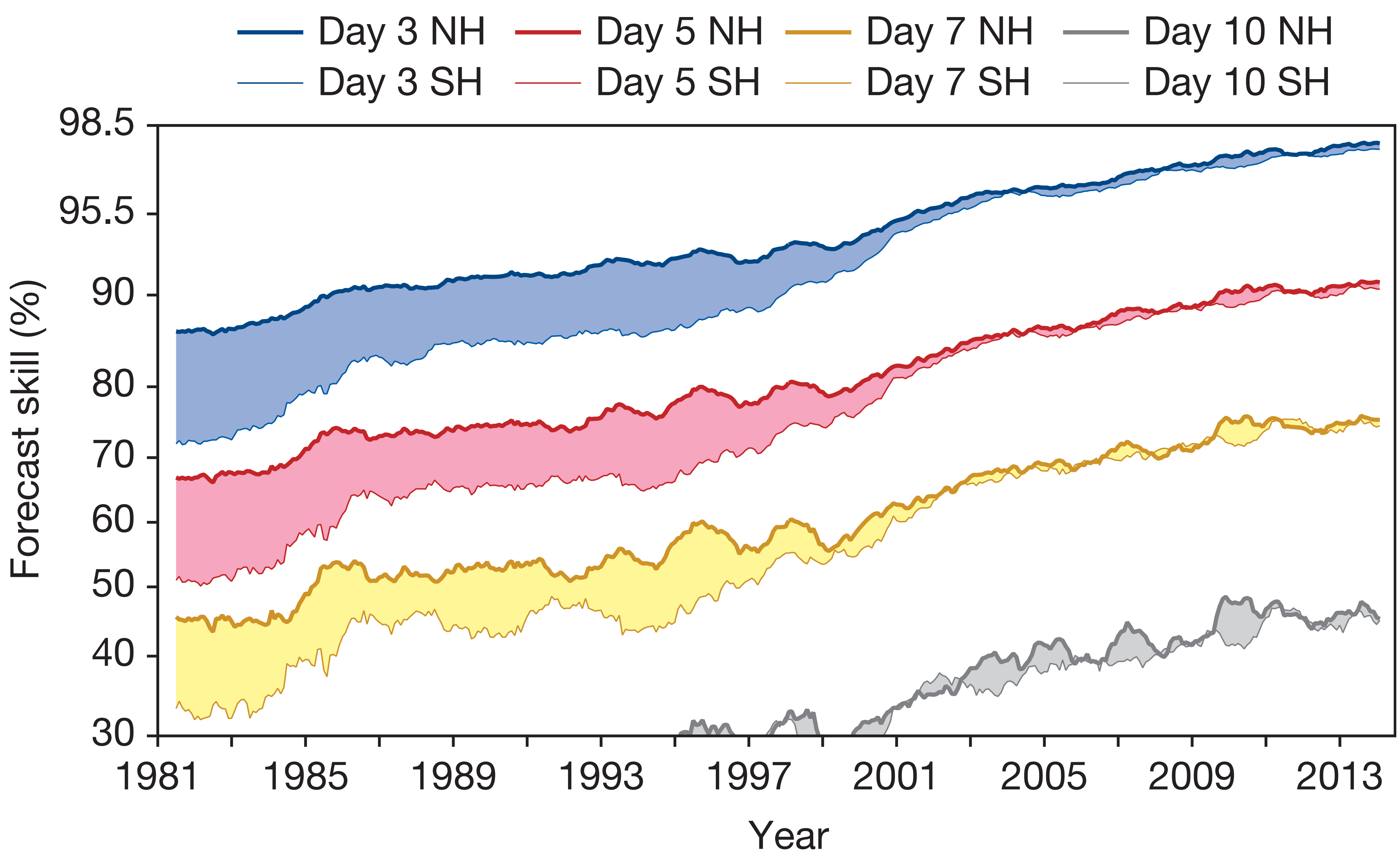
ovember 2008, Vol. 63, No. 11

Peter Lynch[1]
and Owen Lynch[2]

[1]*University College Dublin, Meteorology and Climate Centre, Dublin*
[2]*Dublin Software Laboratory, IBM Ireland*

and John von Neumann (1950; cited below as CFvN). The story of this work was recounted by George Platzman in his Victor P. Starr Memorial Lecture (Platzman, 1979). The atmosphere was treated as a single layer, represented by conditions at the 500 hPa level, modelled by the BVE. This equation, expressing the conservation of absolute

Weather forecasting skill



Bauer et al. (2015)

Main questions

- **Can we predict the evolution of geophysical flows arbitrarily far in time?**
- **If not, what is the predictability limit and what are the processes which determine it?**
- **Are some states (regions of phase space) more predictable than others? Why?**
- **Are operational forecasts close to the predictability limit? How to mitigate the impact of unpredictability in practice?**
- **Can we still make probabilistic predictions beyond the limit of deterministic predictability?**

Outline

I. Fundamental concepts

1. Model error

2. Sensitive dependence on initial condition

3. Different notions of predictability

II. Weather forecasting

1. The limit of predictability for the atmosphere

2. Data assimilation

3. Ensemble forecasting

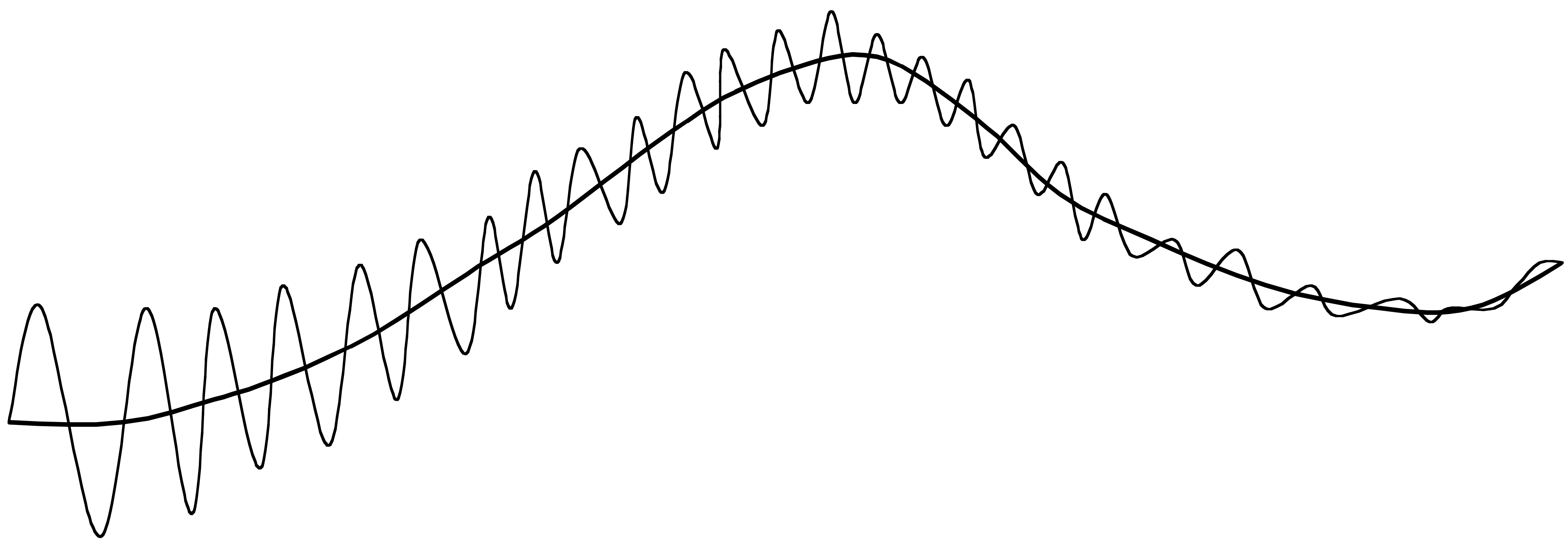
III. Climate Prediction

1. Slow degrees of freedom

2. Sensitive dependence on initial condition

I. Fundamental concepts
1. Model error

Filtering fast oscillations

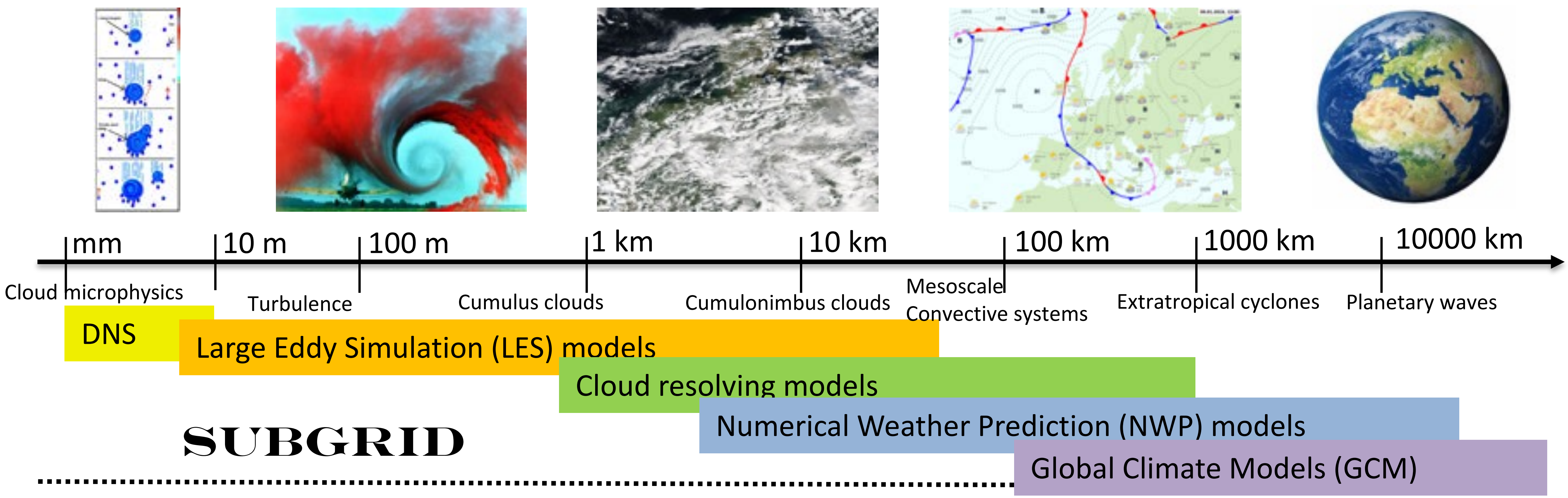


Kalnay, *Atmospheric Modeling, Data Assimilation and Predictability*

Filtered equations

- Fully-compressible Navier-Stokes
- Boussinesq, anelastic -> no sound waves
- quasi-geostrophic -> no internal waves

Unresolved phenomena



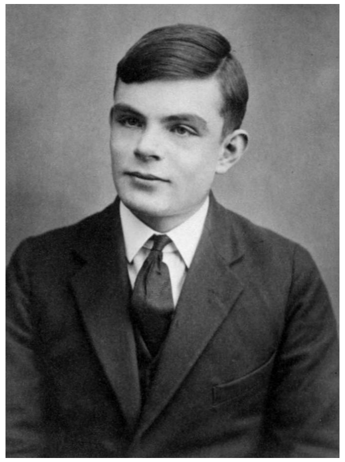
There will always be model error!

Source: N. Vercauteren

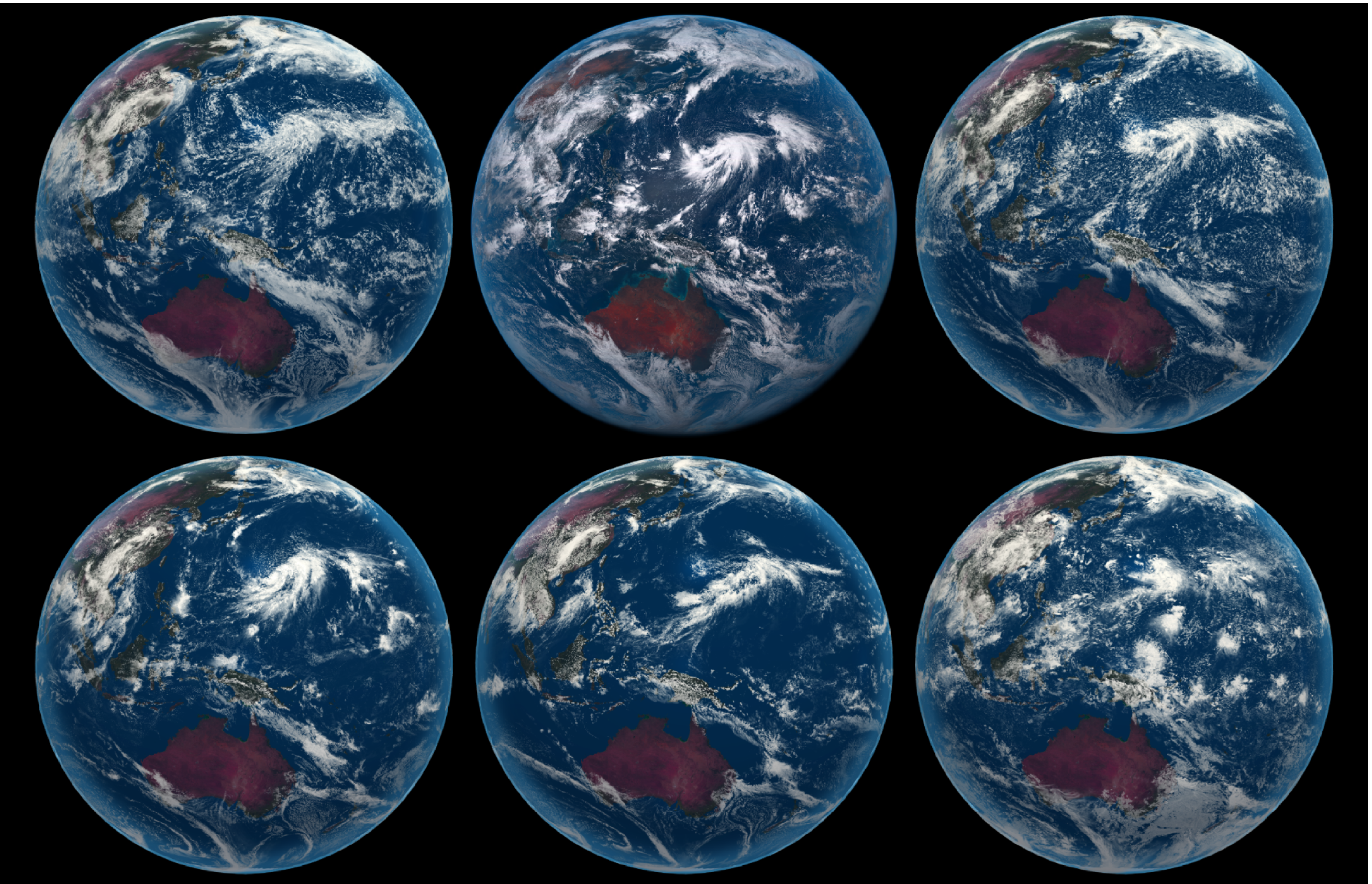
"Turing test" for Climate Models

Palmer (2016)

Can you tell the difference between a human and an artificial intelligence?



1912-1954



Global cloud-resolving models

Stevens et al. (2019)

1. Fundamental concepts
2. Sensitive dependence on initial condition

Sensibilité aux conditions initiales

Pourquoi les météorologistes ont-ils tant de peine à prédire le temps avec quelque certitude ? Pourquoi les chutes de pluie, les tempêtes elles-mêmes nous semblent-elles arriver au hasard, de sorte que bien des gens trouvent tout naturel de prier pour avoir la pluie ou le beau temps, alors qu'ils jugeraient ridicule de demander une éclipse par une prière ? [...] un dixième de degré en plus ou en moins en un point quelconque, le cyclone éclate ici et non pas là, et il étend ses ravages sur des contrées qu'il aurait épargnées. Si on avait connu ce dixième de degré, on aurait pu le savoir à l'avance, mais les observations n'étaient ni assez serrées, ni assez précises, et c'est pour cela que tout semble dû à l'intervention du hasard.

H. Poincaré, *Science et Méthode*, Paris, 1908

The Lorenz equations

E. Lorenz (1963)

130 JOURNAL OF THE ATMOSPHERIC SCIENCES VOLUME 20

Deterministic Nonperiodic Flow[1]

EDWARD N. LORENZ

Massachusetts Institute of Technology

(Manuscript received 18 November 1962, in revised form 7 January 1963)

Deterministic nonperiodic flow

EN Lorenz - Journal of atmospheric sciences, 1963 - journals.ametsoc.org

… When our results concerning the instability of **nonperiodic flow** are applied to the atmosphere, which is ostensibly **nonperiodic**, they indicate that prediction of the sufficiently distant …

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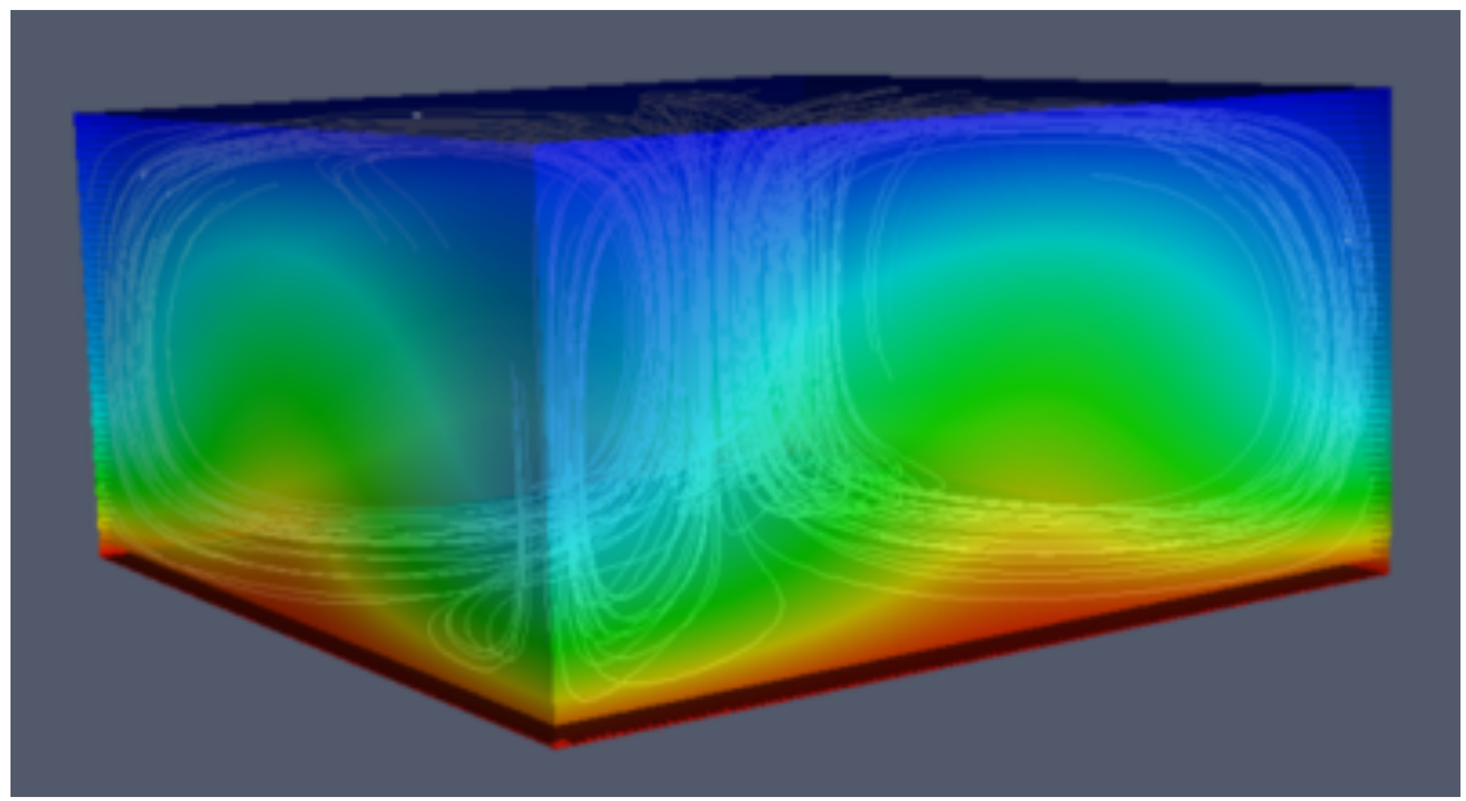
The Lorenz equations

E. Lorenz (1963)

$$\frac{dx}{dt} = \sigma(y - x),$$
$$\frac{dy}{dt} = rx - y - xz,$$
$$\frac{dz}{dt} = xy - bz\,.$$

Spectral truncation of equations for Rayleigh-Bénard convection

Saltzman (1962)

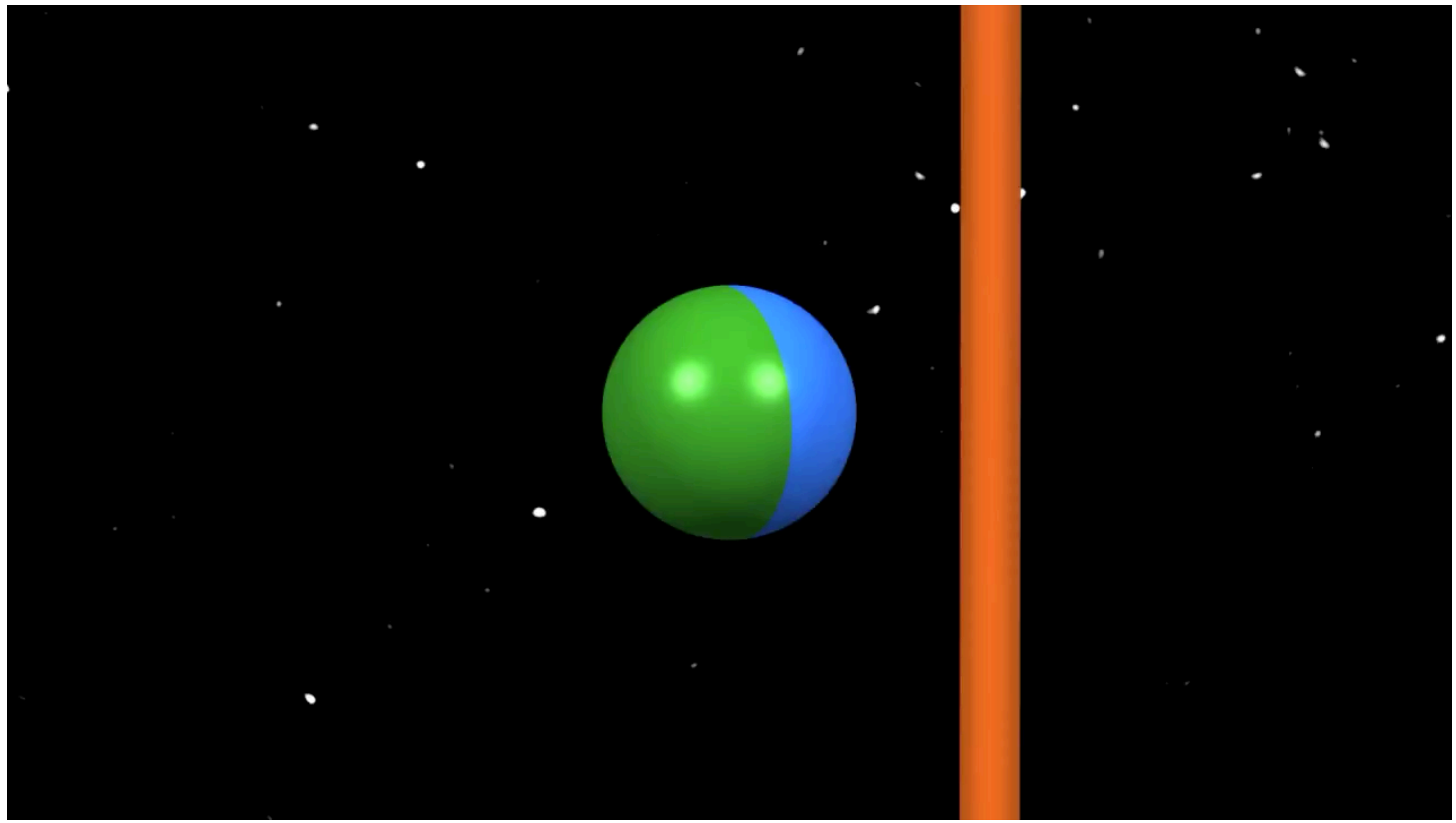


$$\begin{aligned}\psi(x,z,t) &= X(t)\sin x \sin z \\ T(x,z,t) &= Y(t)\cos x \sin z - Z(t)\sin 2z\end{aligned}$$

Sensitive dependence on initial conditions

E. Lorenz (1963)

$$\frac{dx}{dt} = \sigma(y - x),$$
$$\frac{dy}{dt} = rx - y - xz,$$
$$\frac{dz}{dt} = xy - bz\,.$$



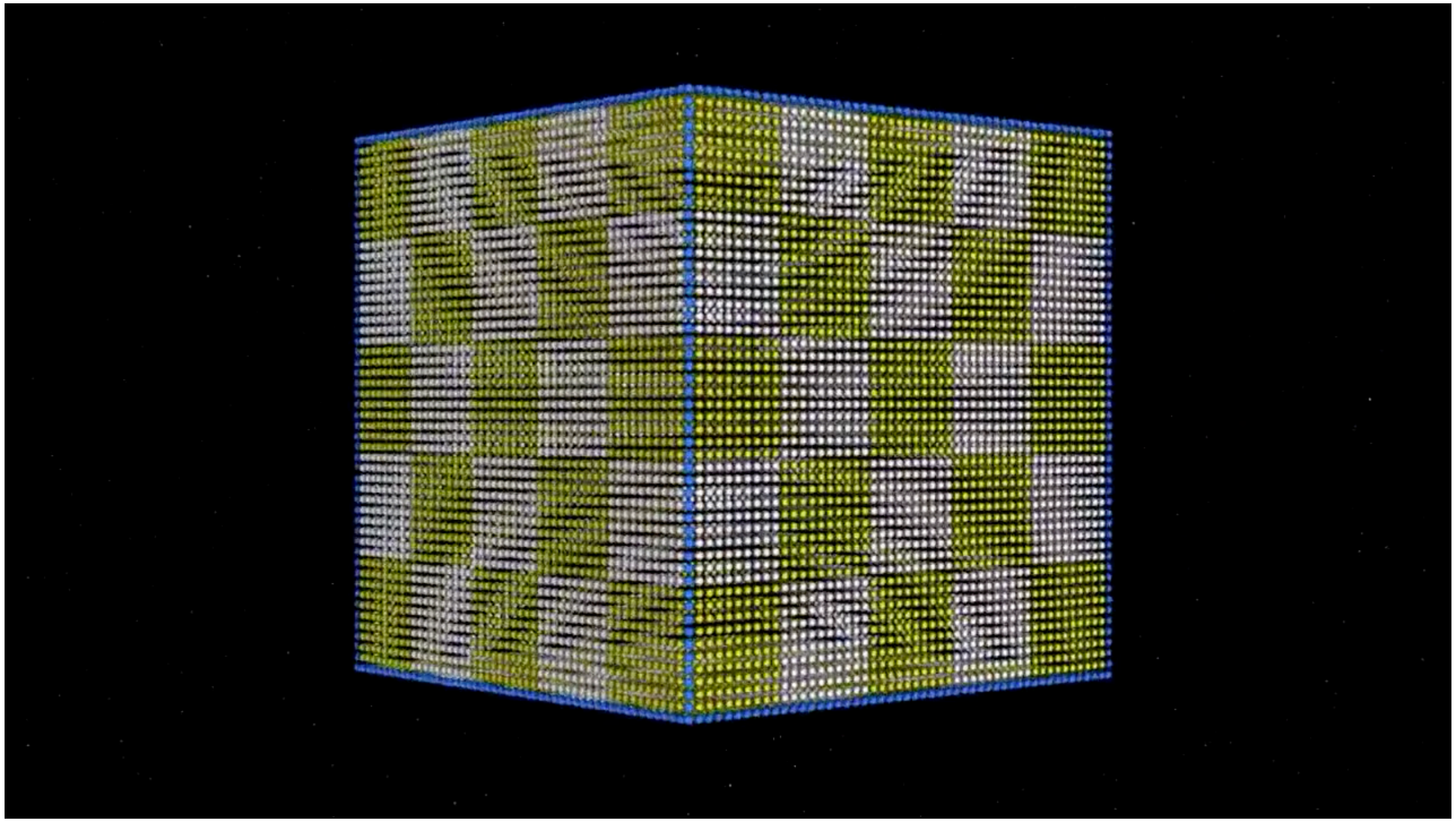
Trajectories with arbitrarily close initial conditions diverge after a finite time.

http://www.chaos-math.org

The Lorenz attractor

E. Lorenz (1963)

$$\frac{dx}{dt} = \sigma(y - x),$$
$$\frac{dy}{dt} = rx - y - xz,$$
$$\frac{dz}{dt} = xy - bz\,.$$



Trajectories with any initial condition converge to a (zero-volume) set in phase space, called "strange attractor".

http://www.chaos-math.org

The Lorenz attractor

E. Lorenz (1963)

$$\frac{dx}{dt} = \sigma(y - x),$$
$$\frac{dy}{dt} = rx - y - xz,$$
$$\frac{dz}{dt} = xy - bz\,.$$

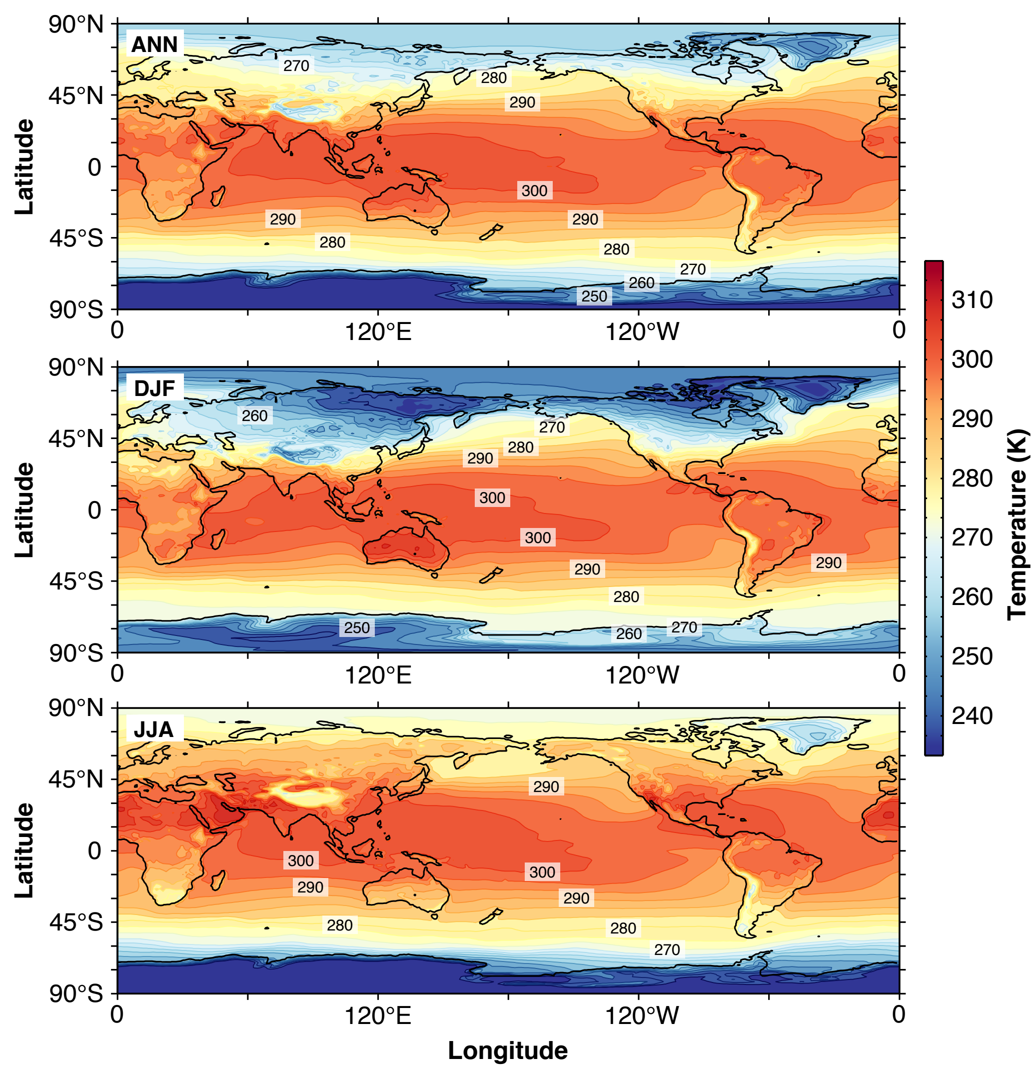
The stationary probability density function does not depend on the initial condition.

http://www.chaos-math.org

I. Fundamental concepts
3. Different notions of predictability

Is climate predictable?

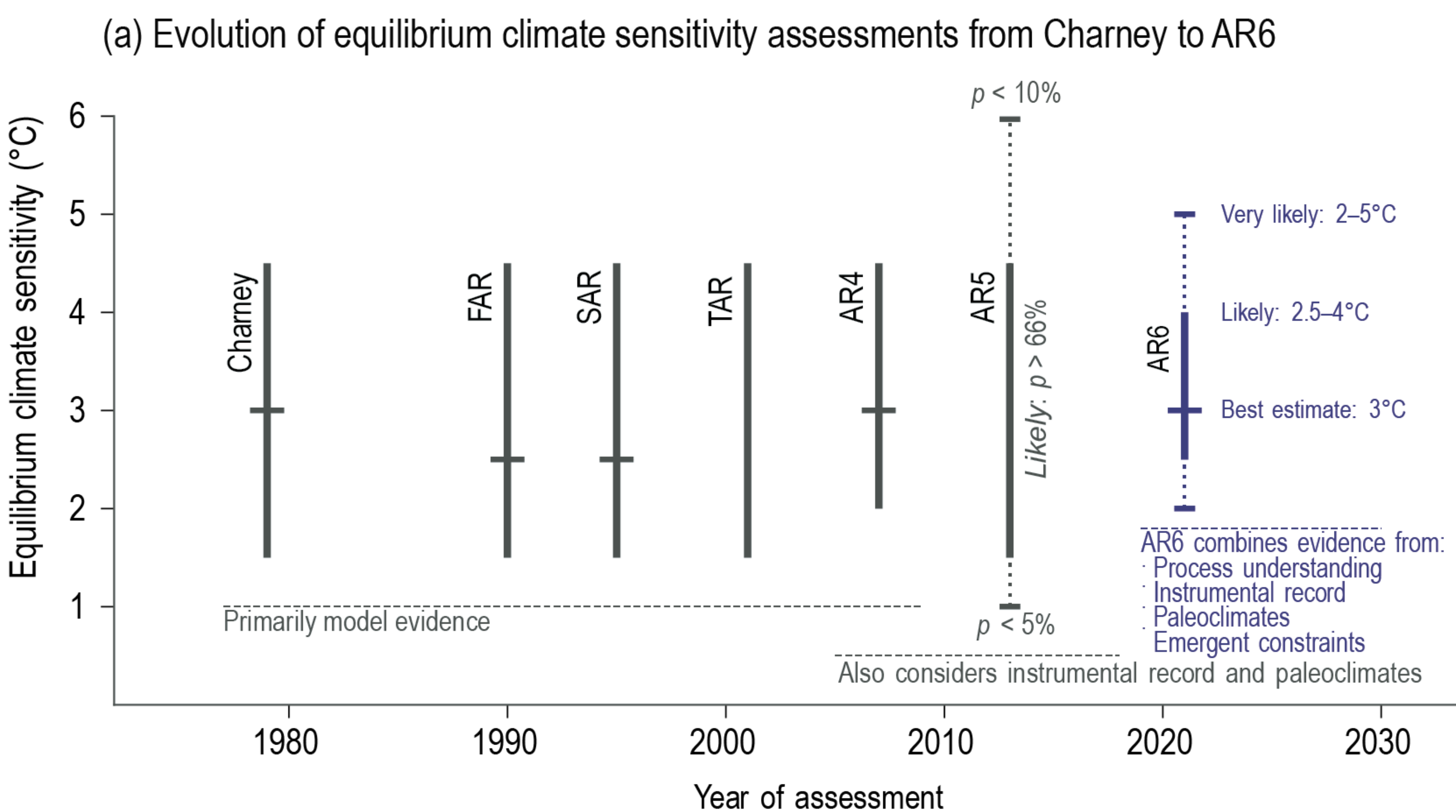
"Climate is what you expect, weather is what you get."



A climate model with different initial conditions gives the same climatology.

Source: T. Schneider

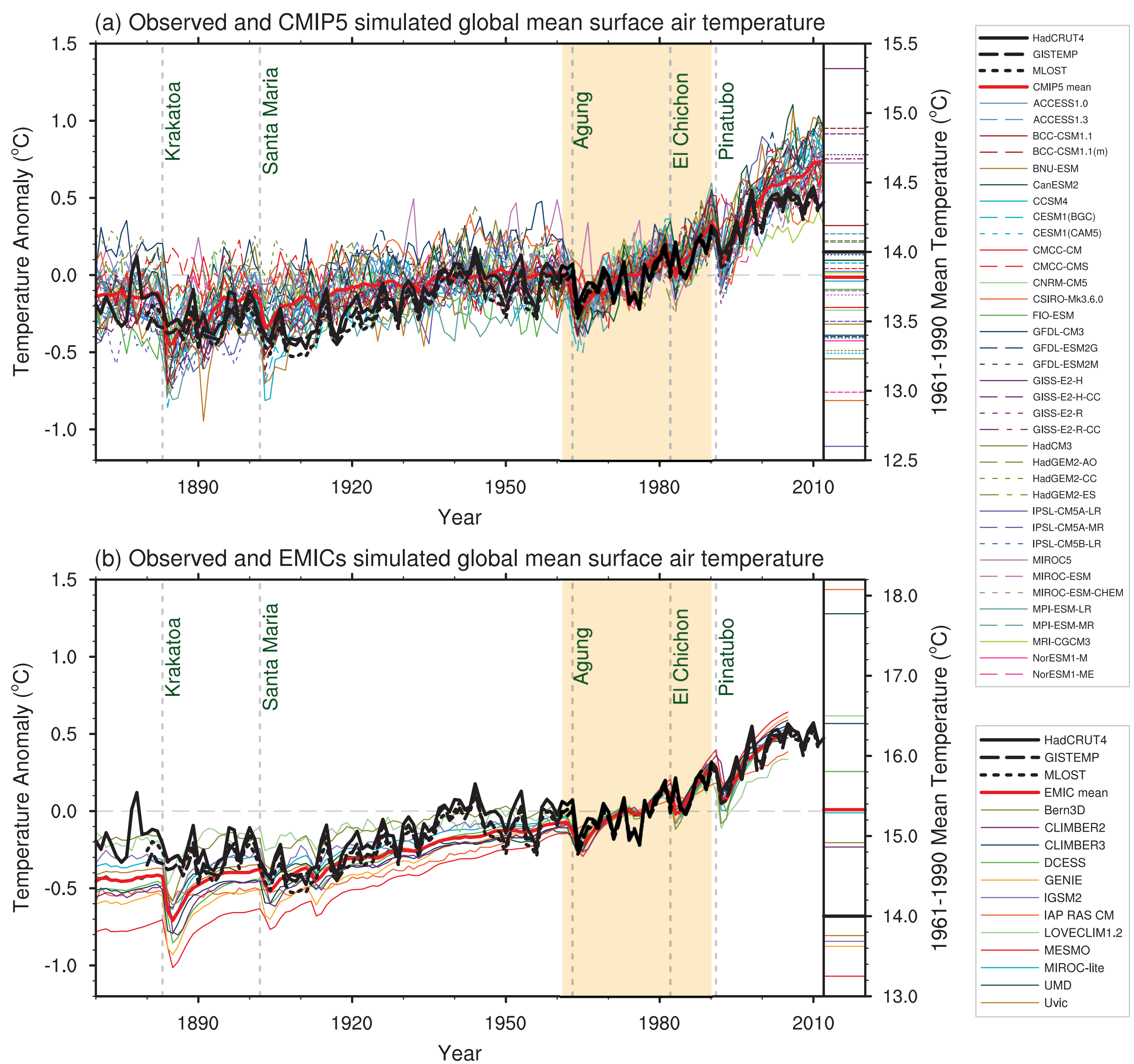
Is climate predictable?



Here the questions we are interested in are of the type: how do the statistics change when some control parameter (e.g. CO_2) changes?

IPCC AR6

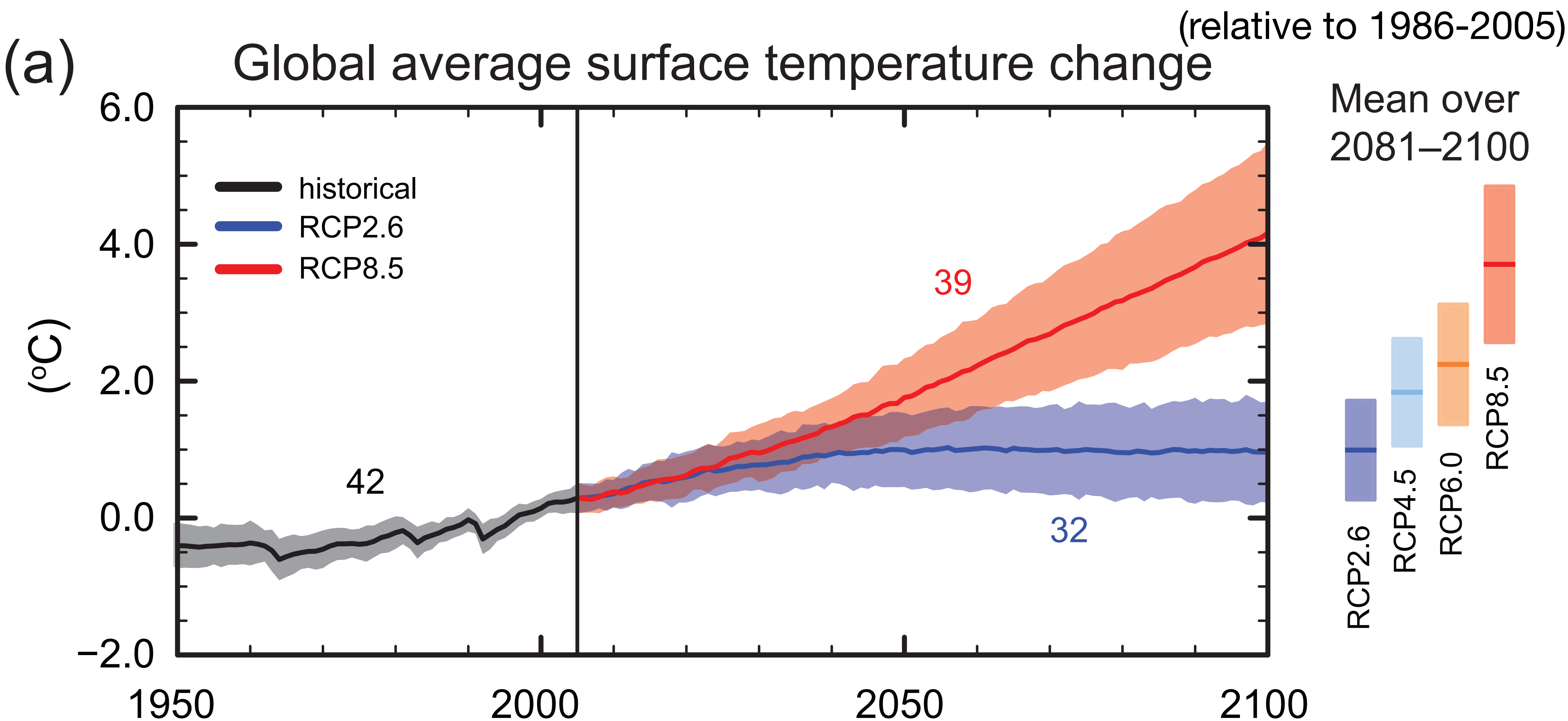
Is climate predictable?



- **The multi-model mean reproduces quite well the historical temperature anomaly**
- **There is significant inter-model variability**

IPCC AR5 Chap. 9

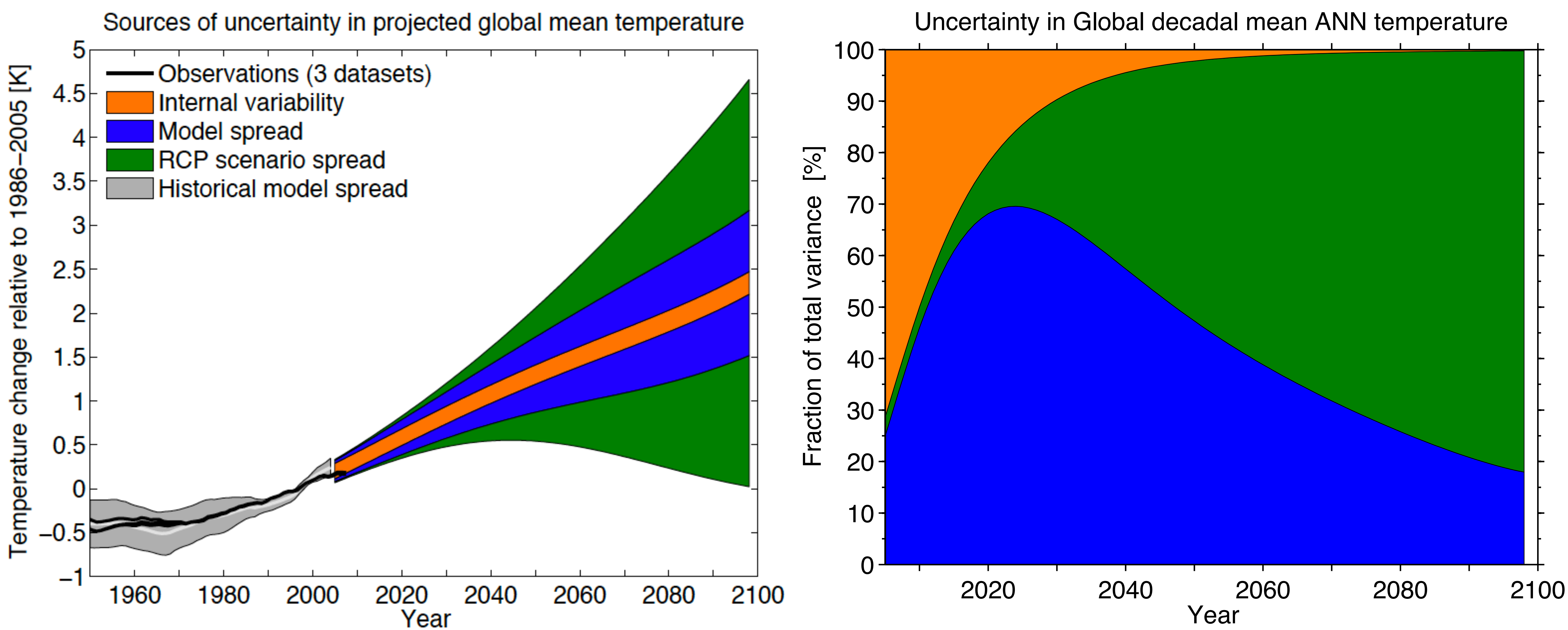
Projections pour la température de surface moyenne



Dans ces projections, le modèle est un outil de prospective.

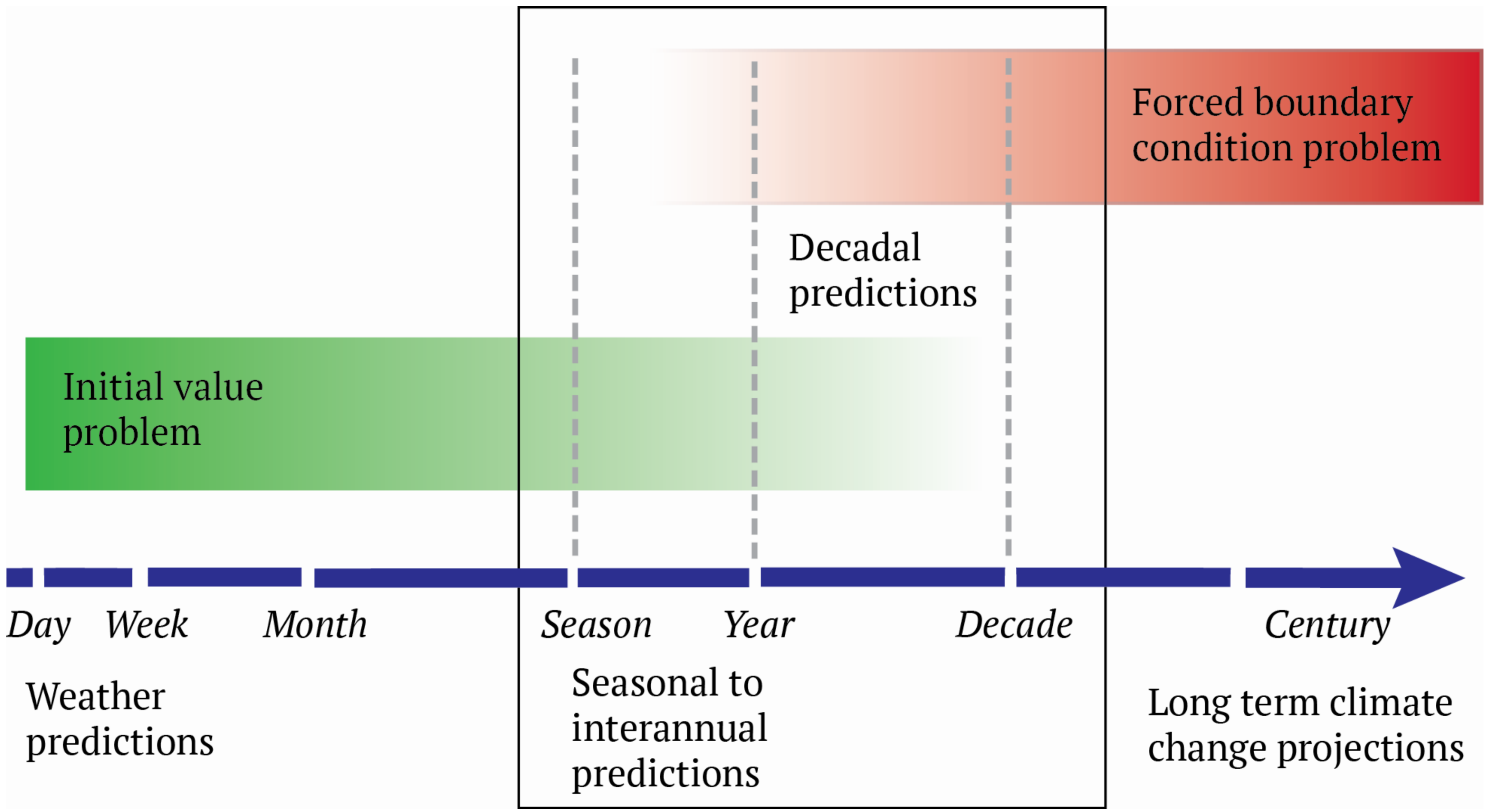
IPCC AR5 SPM

Incertitudes aux différentes échelles de temps



- **Incertitude sur la variabilité interne du système climatique**
  Domine aux temps courts (10 premières années)
- **Incertitude sur la réponse au forçage (e.g. rétroactions due aux nuages)**
  Domine aux temps intermédiaires (entre 10 et 30 ans)
- **Incertitude sur le forçage (e.g. émissions de CO2 à venir)**
  Domine aux temps longs (à partir de 2050)

Different notions of predictability



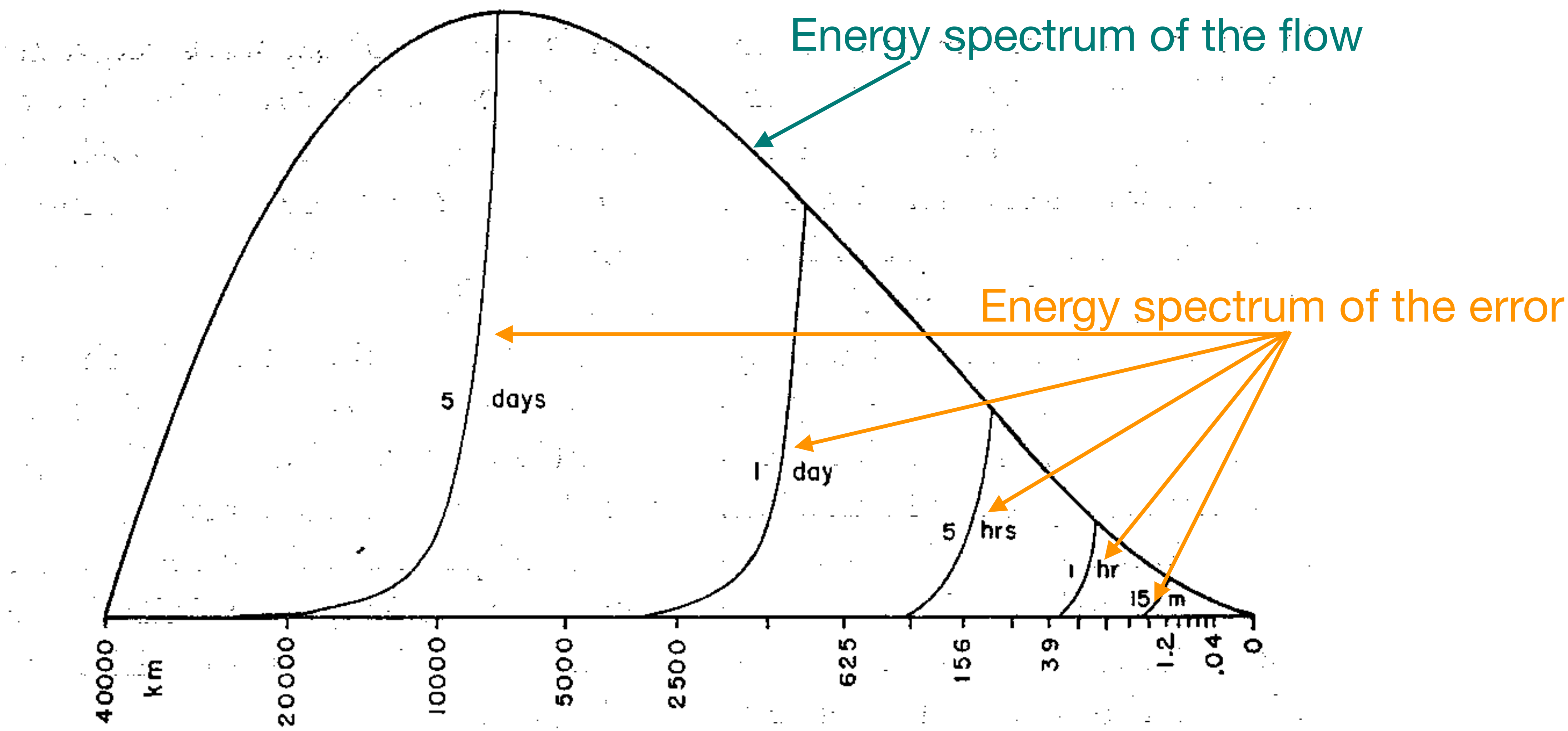
We will first consider problems which are a priori deterministic, like weather forecasting, then intrinsically probabilistic predictions which still depend on the initial condition (climate prediction). We will not discuss climate projections here.

IPCC AR5

II. Weather forecasting
1. The limit of predictability for the atmosphere

Growth-rate of the error

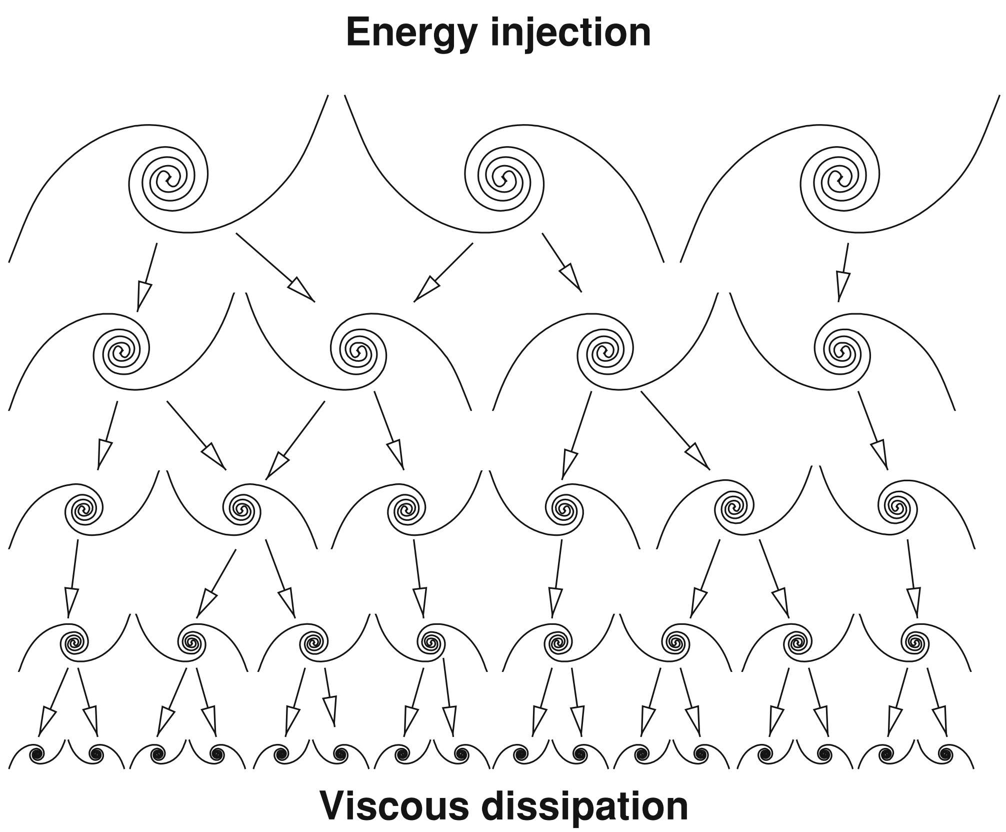
“Dynamical-empirical” approach



- **Each scale of motion has a finite predictability horizon**
- **Halving small-scale error does not appreciably increase large-scale predictability**

Lorenz, *Tellus* (1969)

Error growth in a turbulent cascade



- **Error initially confined to small scales: $k_i >> 1$**
- **Error growth through local (in scale) nonlinear interactions:** time to propagate from k to 2k is the *eddy-turnover time*

$$\tau_{NL} = [k^3 E(k)]^{-1/2}$$

- **Total time to reach scale k_f:**

$$T = \int_{k_f}^{k_1} [k^3 E(k)]^{1-/2} d(\ln k)$$

- **3D homogeneous isotropic turbulence:** **Finite predictability!**

$$E(k) = C_K \varepsilon^{2/3} k^{-5/3}, \quad T \sim \varepsilon^{-1/3} \left(k_f^{-2/3} - k_i^{-2/3} \right) \rightarrow \varepsilon^{-1/3} k_f^{-2/3} \text{ as } k_i \rightarrow +\infty$$

- **2D turbulence (enstrophy cascade):**

$$E(k) = C \eta^{2/3} k^{-3}, \quad T \sim \eta^{-1/3} \ln \left(k_i / k_f \right) \rightarrow +\infty \text{ as } k_i \rightarrow +\infty$$

Error growth through non-local interaction

Small-scale error (amplitude A_i at wave number k_i) interacts directly with large-scale field. The large-scale error grows exponentially; assume the growth rate is the inverse of the large-eddy turnover time.

$$A_f \sim A_i e^{t/\tau_{NL}(k_f)}, \text{ with } A \sim E(k)$$

The error saturates when

$$E(k_f) \sim E(k_i) e^{T/\tau_{NL}(k_f)}$$

- **3D homogeneous isotropic turbulence:**

$$E(k) = C_K \varepsilon^{2/3} k^{-5/3}, \quad T \sim \varepsilon^{-1/3} k_f^{-2/3} \ln(k_i/k_f)$$

- **2D turbulence (enstrophy cascade):**

$$E(k) = C\eta^{2/3} k^{-3}, \quad T \sim \eta^{-1/3} \ln\left(k_i/k_f\right)$$

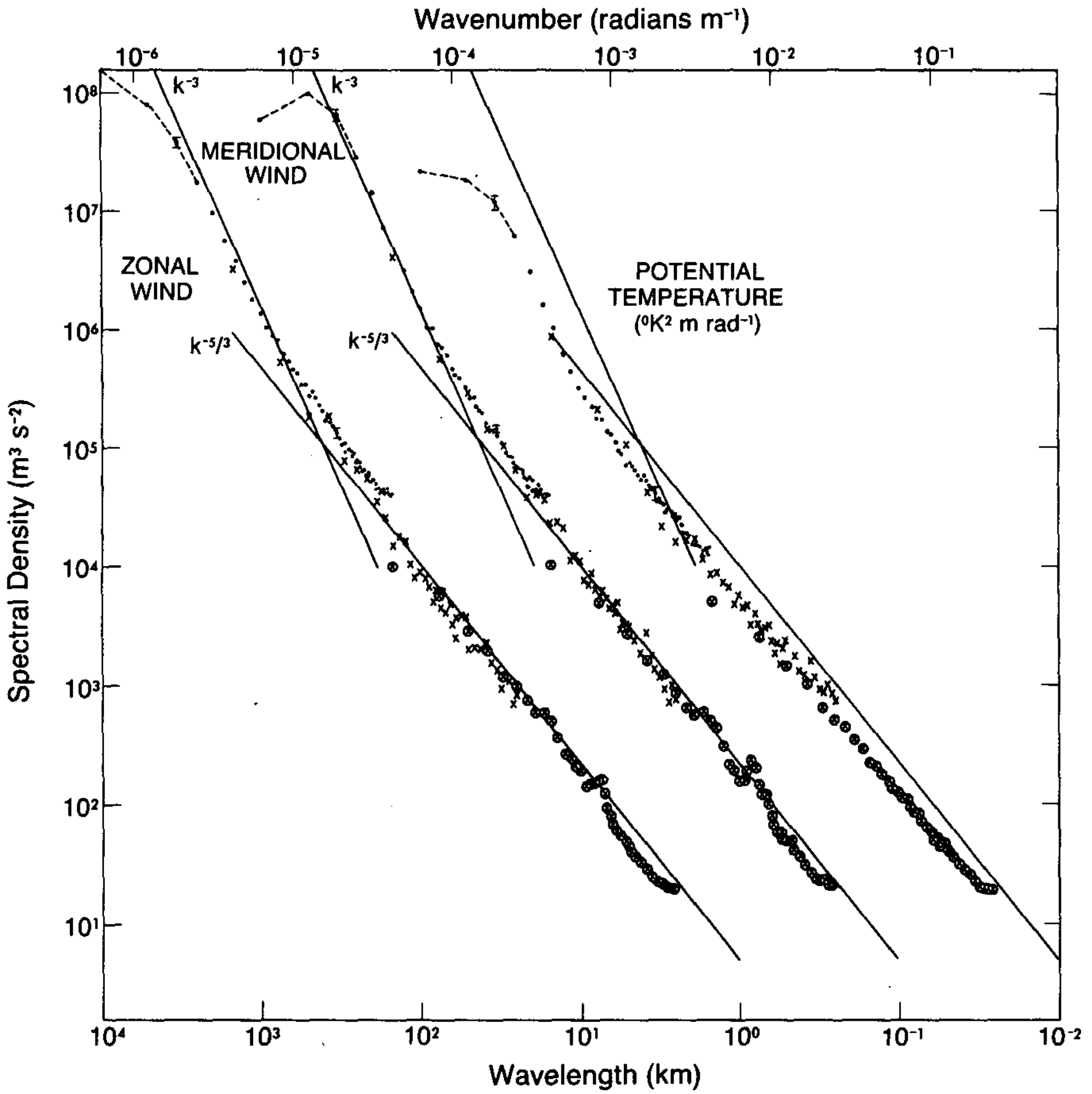
Cascade mechanism dominates in 3D, not necessarily in 2D (scale-independent eddy-turnover time)

Application to the Atmosphere

Can we estimate the order of magnitude of the limit or predictability of the atmosphere?

- **Error initially at infinitely small scales**
- **Time to reach 100 km (3D turbulence): ~0.5 day**
- **Time to reach 1000 km: ~2 days**

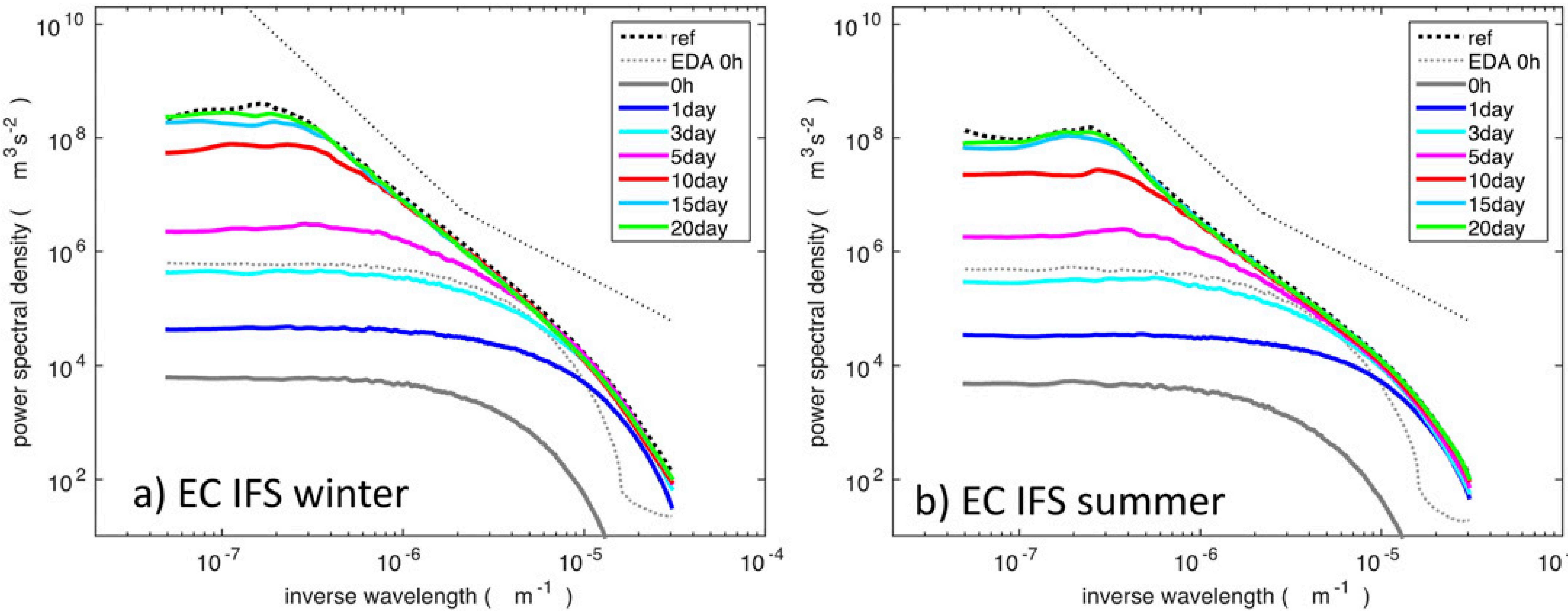
Compatible with the estimate of Lorenz



Nastrom et al. (1984)

Error growth in atmospheric models

For NH mid-latitudes



- **Qualitative error growth mechanism compatible with cascade mechanism**
- **Order of magnitude of timescales similar to simple estimate (a bit longer, probably because the spectrum is steeper at small scales)**

Zhang et al. (2019)

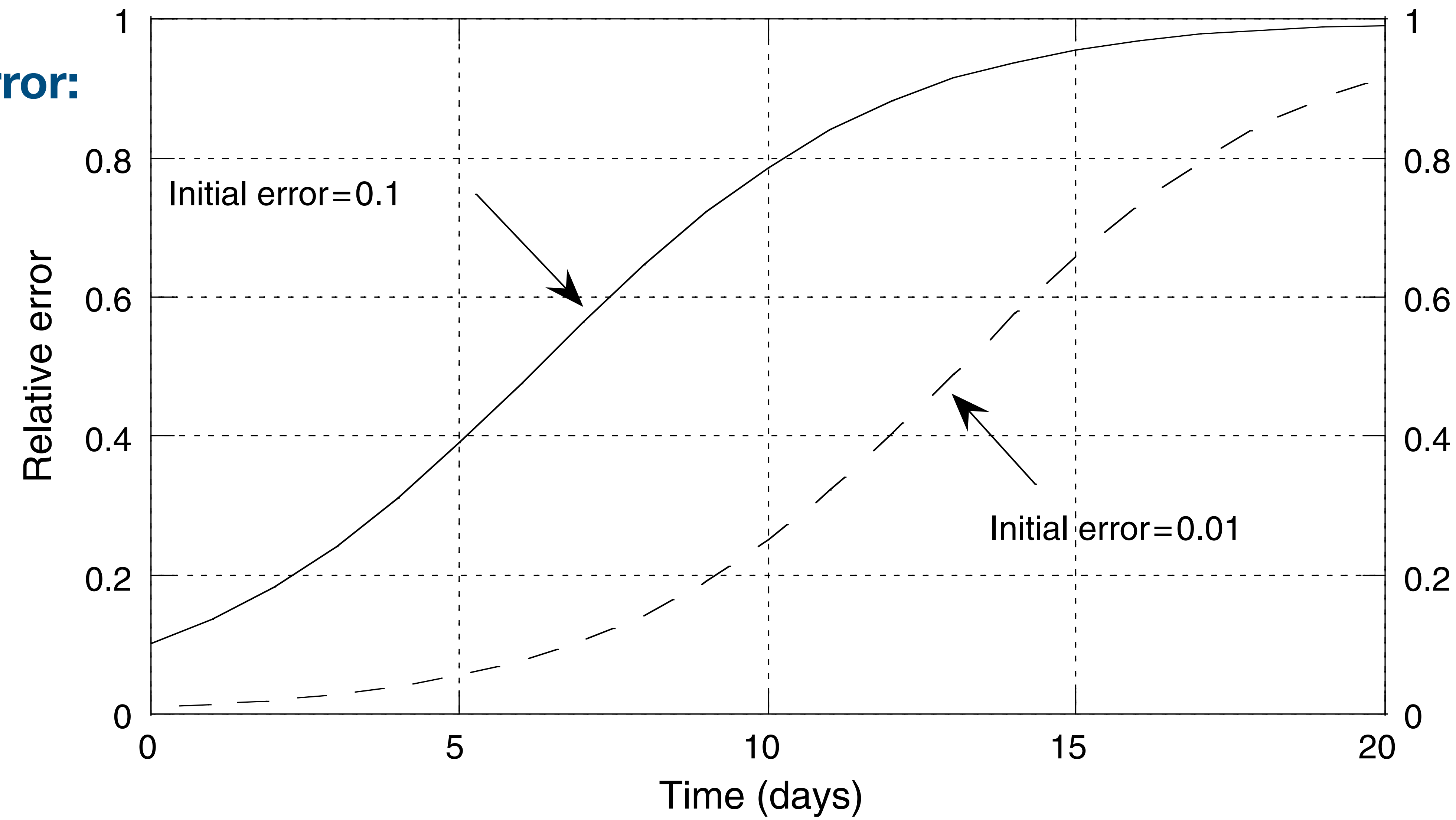
Predictability limit of the mid-latitudes

Simple model for the error: logistic equation

$$\frac{d\varepsilon}{dt} = a\varepsilon(1-\varepsilon)$$

$$\varepsilon = \frac{\varepsilon_0 e^{at}}{1+\varepsilon_0(e^{at}-1)}$$

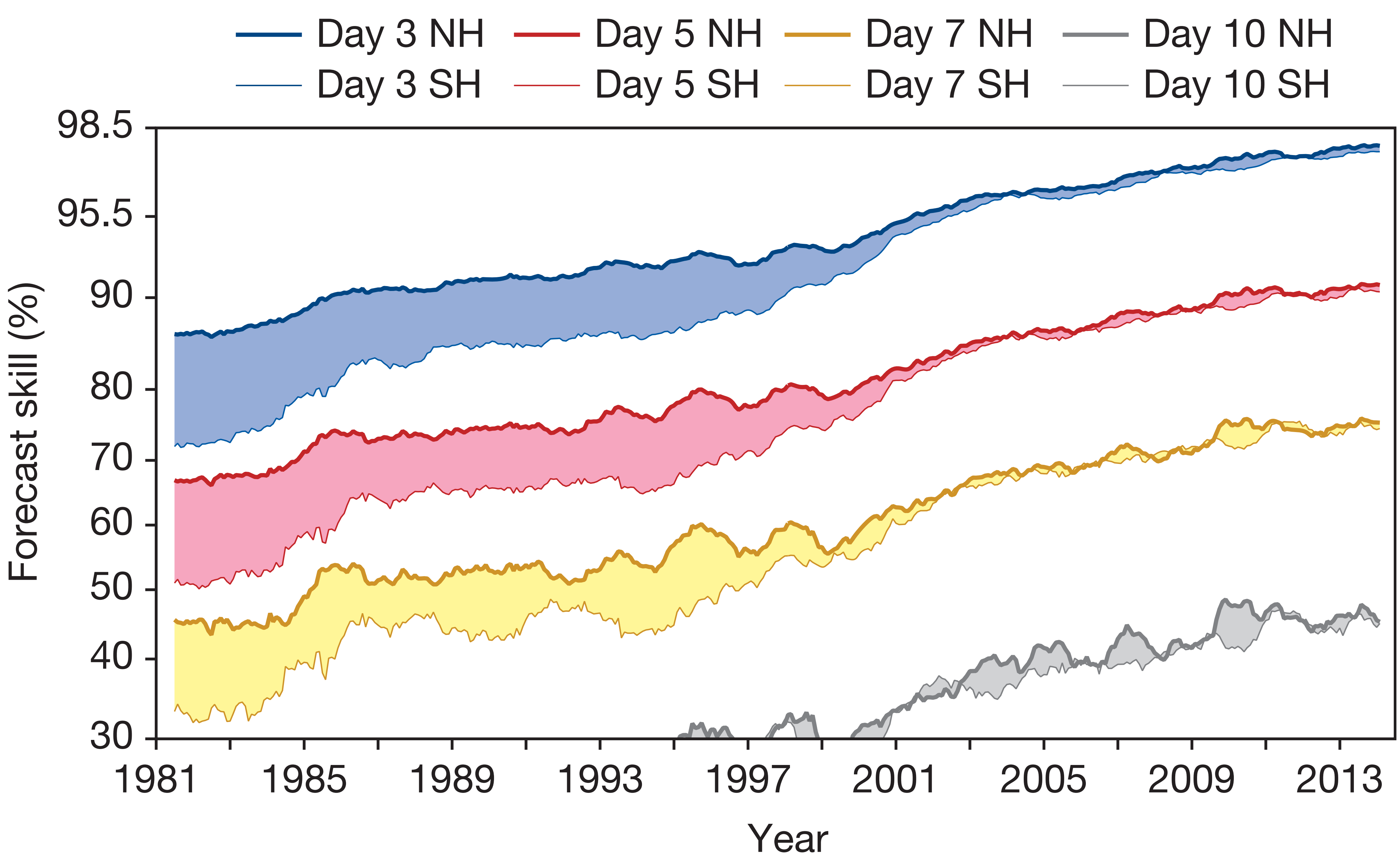
$$a = 0.35\ \textbf{day}^{-1}$$



Predictability limit around 15 days

Kalnay, *Atmospheric Modeling, Data Assimilation and Predictability*

Evolution of Numerical Weather Prediction



Bauer et al. *Nature* (2015)

State-dependent predictability

As we shall see, predictability properties depends on the state of the atmosphere at the time of prediction.

The predictability depends on the local instabilities of the flow in phase space.

How to study this quantitatively?

- **Singular vectors of the tangent linear model**
- **Lyapunov vectors**

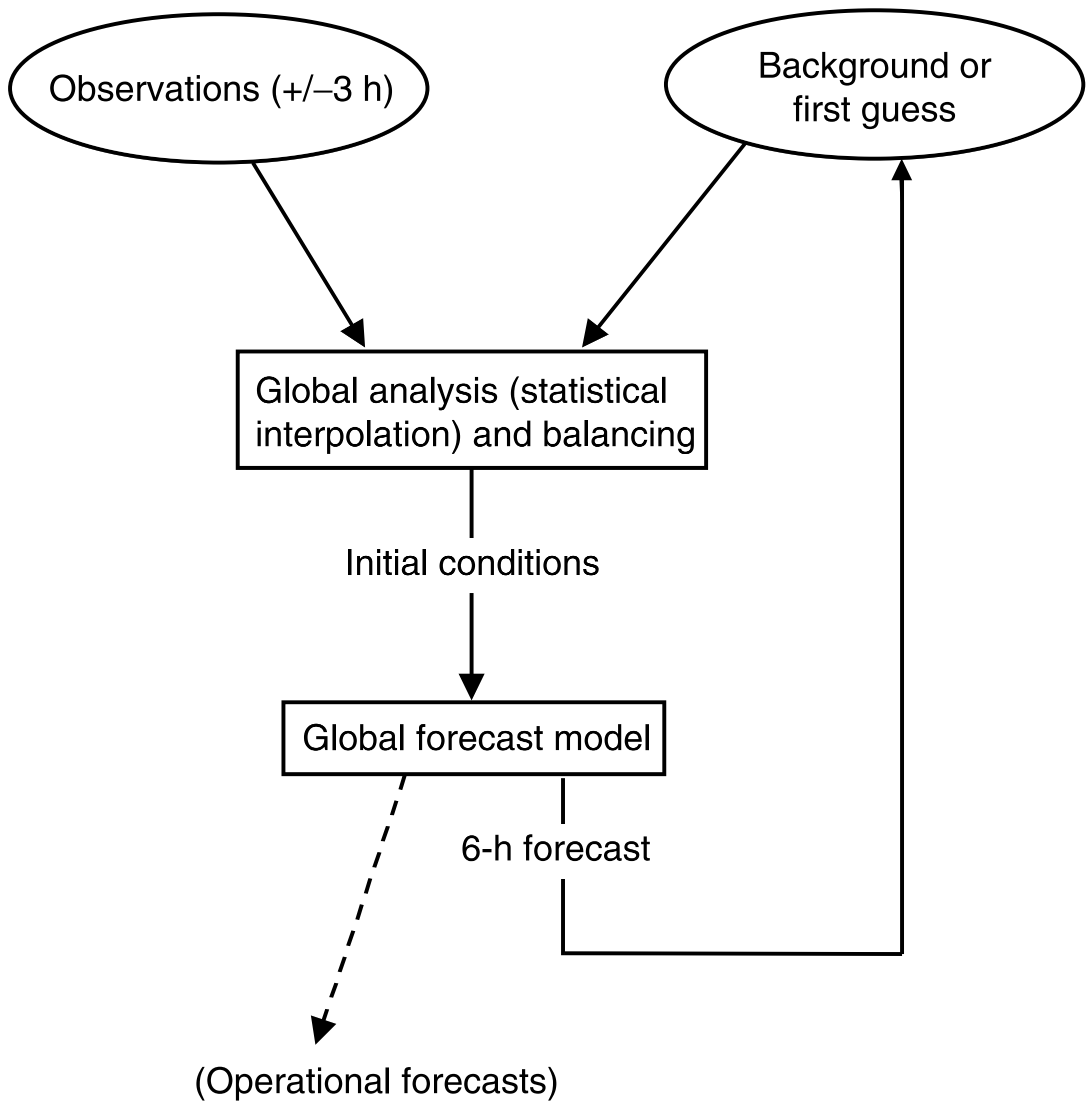
See e.g. Kalnay, *Atmospheric Modeling, Data Assimilation and Predictability*

II. Weather Forecasting
2. Data Assimilation

How to determine the initial condition for a prediction?

$$\dot{X} = F(X), \quad X \in \mathbb{R}^n$$

The dimension of the model space is much larger than the number of available observations: inverse problem!



Kalnay, *Atmospheric Modeling, Data Assimilation and Predictability*

Optimal interpolation (simple)

Suppose you have two estimates for a quantity, what is the optimal combination?

$$\hat{T}_b = T + \varepsilon_b, \qquad \mathbb{E}[\varepsilon_b] = 0, \qquad \mathbb{E}[\varepsilon_b^2] = \sigma_b^2$$ (model prediction="background")

$$\hat{T}_o = T + \varepsilon_o, \qquad \mathbb{E}[\varepsilon_o] = 0, \qquad \mathbb{E}[\varepsilon_o^2] = \sigma_o^2$$ (observation)

$$\mathbb{E}[\varepsilon_b \varepsilon_o] = 0$$

Optimal linear interpolation: $\hat{T}_a = a_b \hat{T}_b + a_o \hat{T}_o$ (analysis)

- **Unbiased estimator:** $\mathbb{E}[\hat{T}_a] = T$ implies $a_b + a_o = 1$
- **Minimize mean-square error:** $\sigma_a^2 = \mathbb{E}[(\hat{T}_a - T)^2] = \mathbb{E}[(a_b \varepsilon_b + a_o \varepsilon_o)^2]$

Solution: $a_b = \dfrac{\sigma_o^2}{\sigma_b^2 + \sigma_o^2}, \quad a_o = \dfrac{\sigma_b^2}{\sigma_b^2 + \sigma_o^2}$

$$\hat{T}_a = \hat{T}_b + W(\hat{T}_o - \hat{T}_b), \quad W = a_o,$$ analysis = background+gain*innovation

$$\sigma_a^2 = (1 - W)\sigma_b^2$$

Optimal interpolation (general)

This time the analysis, background and observation vectors can have arbitrary dimensions, and the observations are indirect.

Error covariance matrices:

$$\mathbf{A} = \mathbb{E}[\boldsymbol{\varepsilon}_{\boldsymbol{a}}\boldsymbol{\varepsilon}_a^T],$$
$$\mathbf{B} = \mathbb{E}[\boldsymbol{\varepsilon}_b\boldsymbol{\varepsilon}_b^T],$$
$$\mathbf{R} = \mathbb{E}[\boldsymbol{\varepsilon}_o\boldsymbol{\varepsilon}_o^T]$$

We assume $\mathbb{E}[\boldsymbol{\varepsilon}_o\boldsymbol{\varepsilon}_b^T] = 0$

Linearization of "forward observational operator":

$$H(\mathbf{x} + \delta\mathbf{x}) = H(\mathbf{x}) + \mathbf{H}\delta\mathbf{x} + o(\delta\mathbf{x}),$$
$$\mathbf{y}_o - H(\mathbf{x}_b) = \boldsymbol{\varepsilon}_o - \mathbf{H}\boldsymbol{\varepsilon}_b$$

We assume that the background (model) and observation error are unbiased:

$$\mathbb{E}[\boldsymbol{\varepsilon}_b] = \mathbb{E}[\boldsymbol{\varepsilon}_0] = 0$$

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{W}[\mathbf{y}_o - H(\mathbf{x}_b)],$$
$$\mathbf{W} = \mathbf{B}\mathbf{H}^T(\mathbf{R} + \mathbf{H}\mathbf{B}\mathbf{H}^T)^{-1},$$
$$\mathbf{A} = (\mathbf{I} - \mathbf{W}\mathbf{H})\mathbf{B}$$

To use this formula in practice we need to estimate B, R and H.

Kalman Filter

The idea of the Kalman filter is to use iteratively the optimal interpolation method to propagate both the state of the system and the error covariance matrix.

$$
\begin{aligned}
\mathbf{x}_b^{n+1} &= \mathbf{M}(\mathbf{x}_a^n), \\
\mathbf{x}_a^n &= \mathbf{x}_b^n + \mathbf{W}^n[\mathbf{y}_o^n - H(\mathbf{x}_b^n)], \\
\mathbf{W}^n &= \mathbf{B}^n\mathbf{H}^T(\mathbf{R} + \mathbf{H}\mathbf{B}^n\mathbf{H}^T)^{-1}, \\
\mathbf{B}^{n+1} &= \mathbf{L}_n\mathbf{A}^n\mathbf{L}_n^T, \\
\mathbf{A}^n &= (\mathbf{I} - \mathbf{W}^n\mathbf{H})\mathbf{B}^n
\end{aligned}
$$

with L_n the tangent linear model.

A new approach to linear filtering and prediction problems

RE Kalman - 1960 - asmedigitalcollection.asme.org

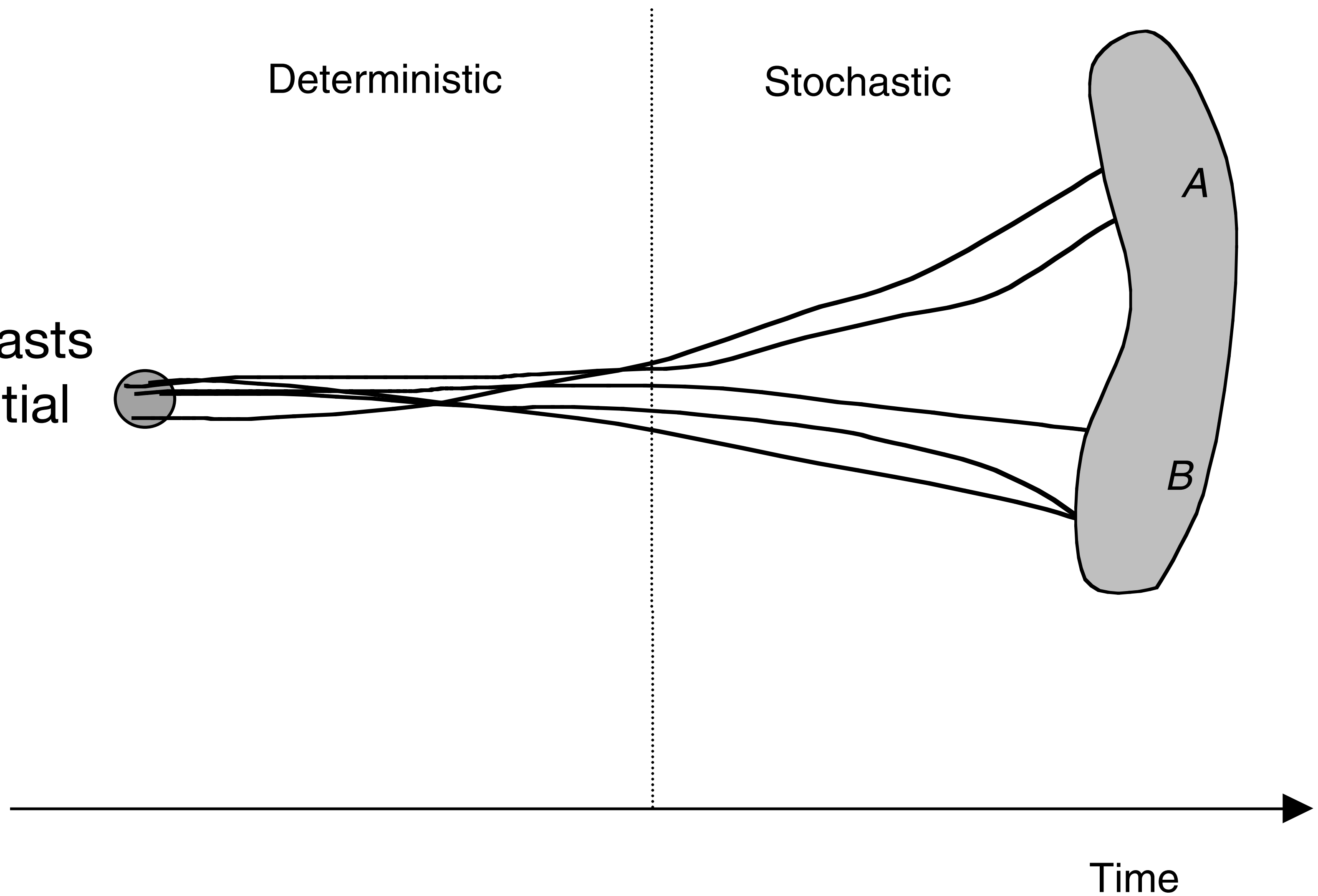
The classical filtering and prediction problem is re-examined using the Bode-Shannon representation of random processes and the "state-transition" method of analysis of dynamic …

The Kalman filter becomes prohibitively expensive for high-dimensional systems, so in practice some approximations are made (e.g. using ensemble methods).

II. Weather Forecasting
3. Ensemble Forecasting

Goals of ensemble forecasting

Ensemble of forecasts with perturbed initial conditions



- **Quantify the forecast error**
- **Quantify the predictability**

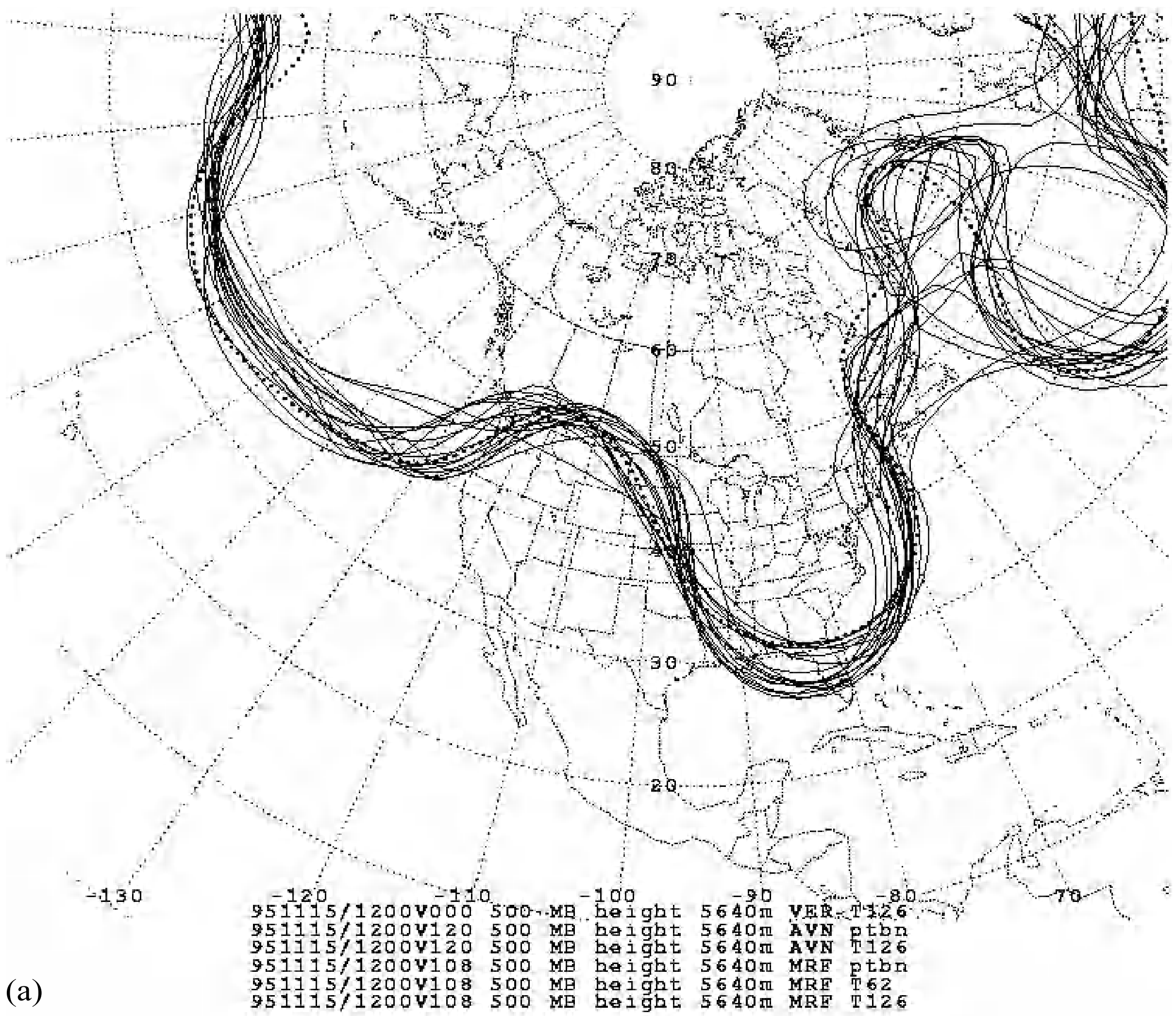
Kalnay, *Atmospheric Modeling, Data Assimilation and Predictability*

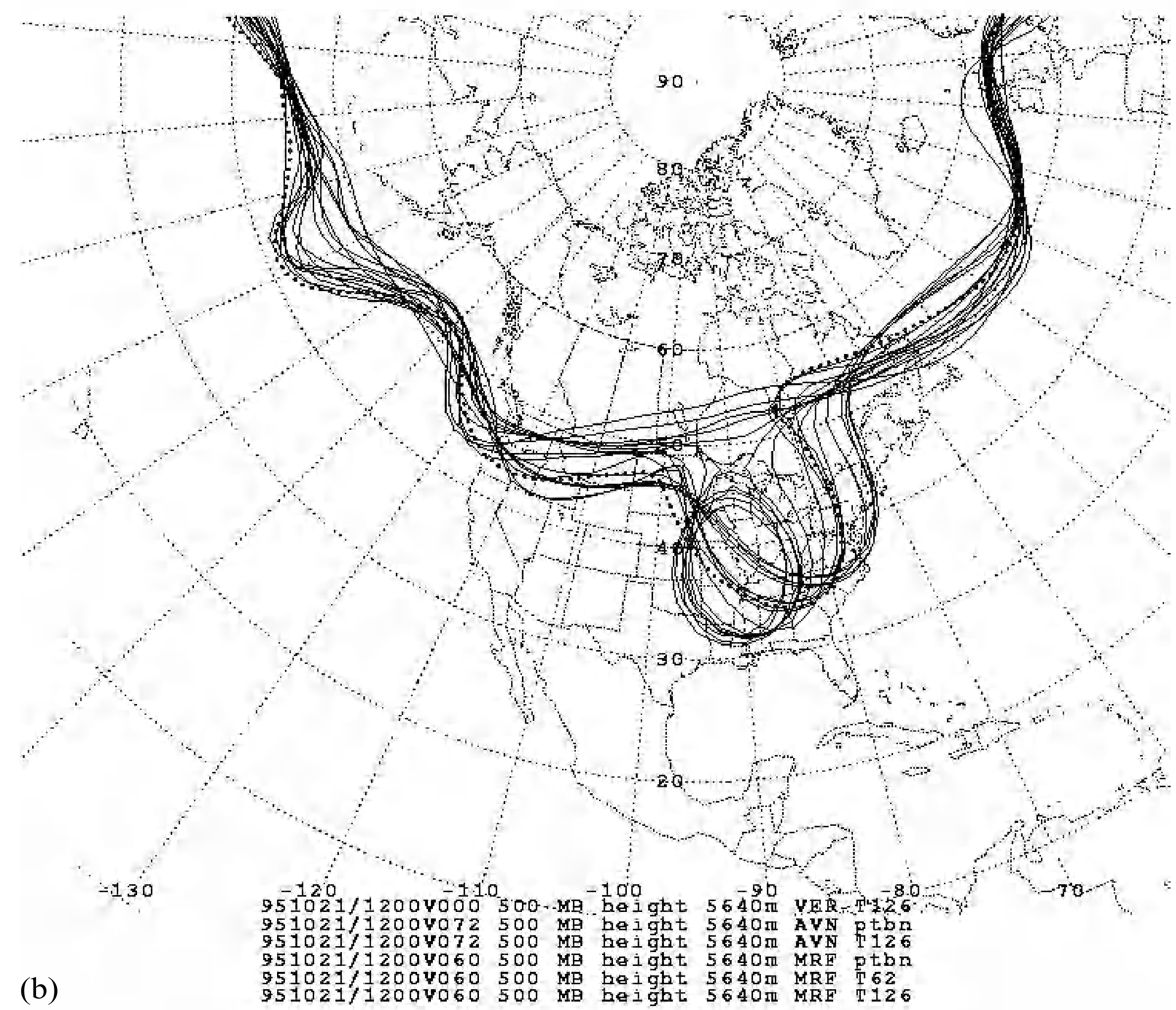
Example of ensemble forecast

17 ensemble members

5-day forecast for 15 Nov 1995 (NCEP)

2.5-day forecast for 21 Oct 1995 (NCEP)





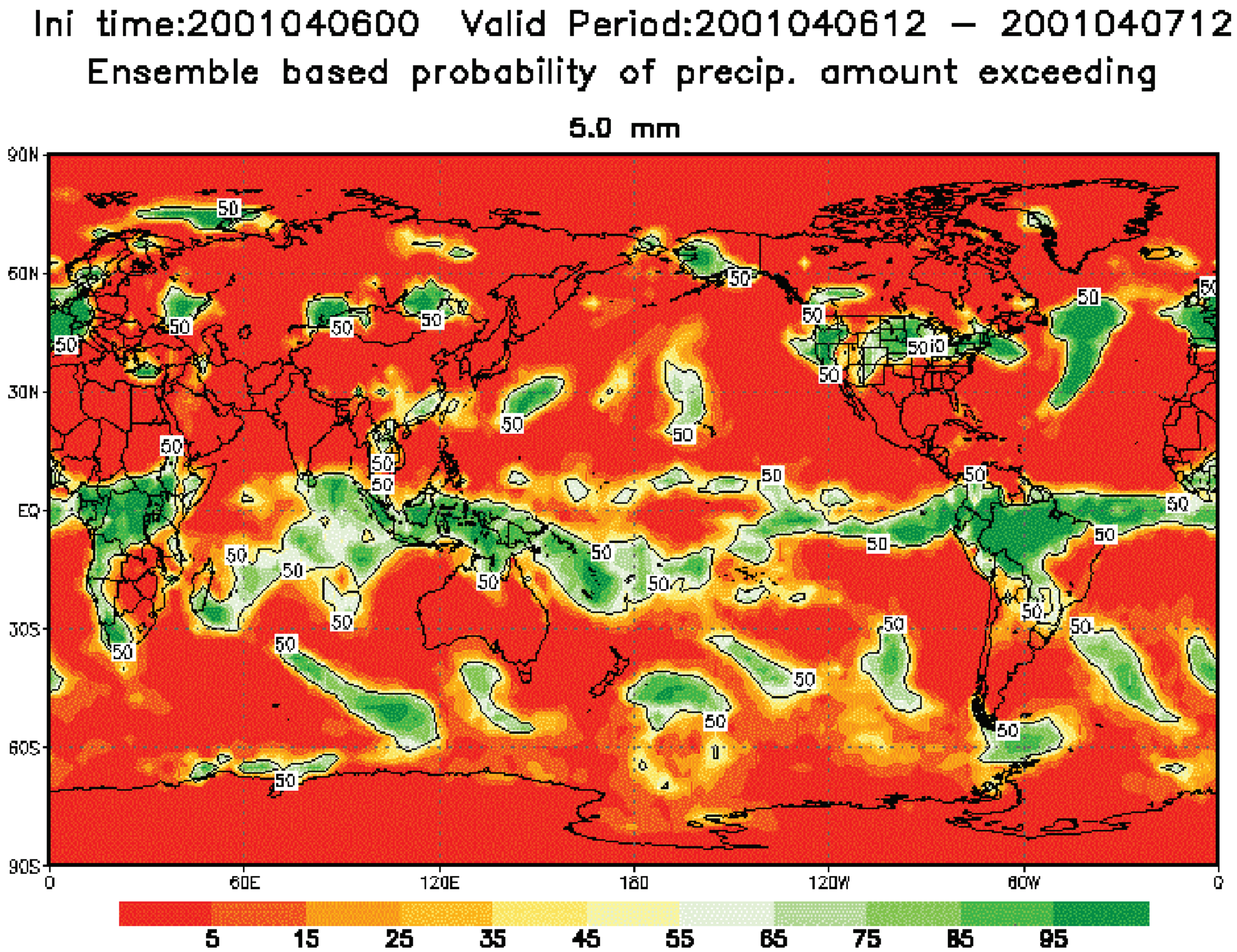
Predictable winter storm

Limited predictability

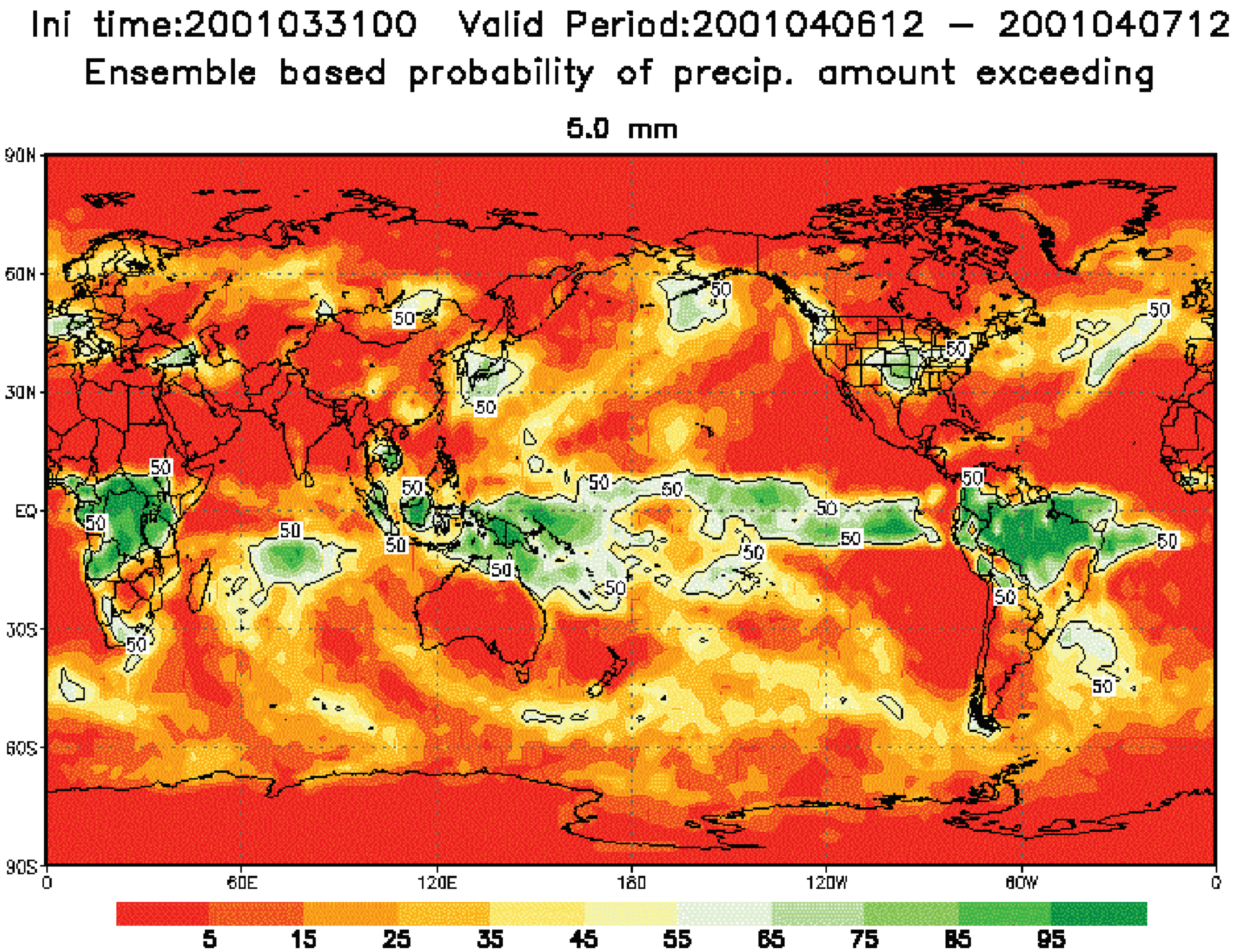
Kalnay, *Atmospheric Modeling, Data Assimilation and Predictability*

Example of ensemble forecast

Fraction of ensemble members with P > 5mm



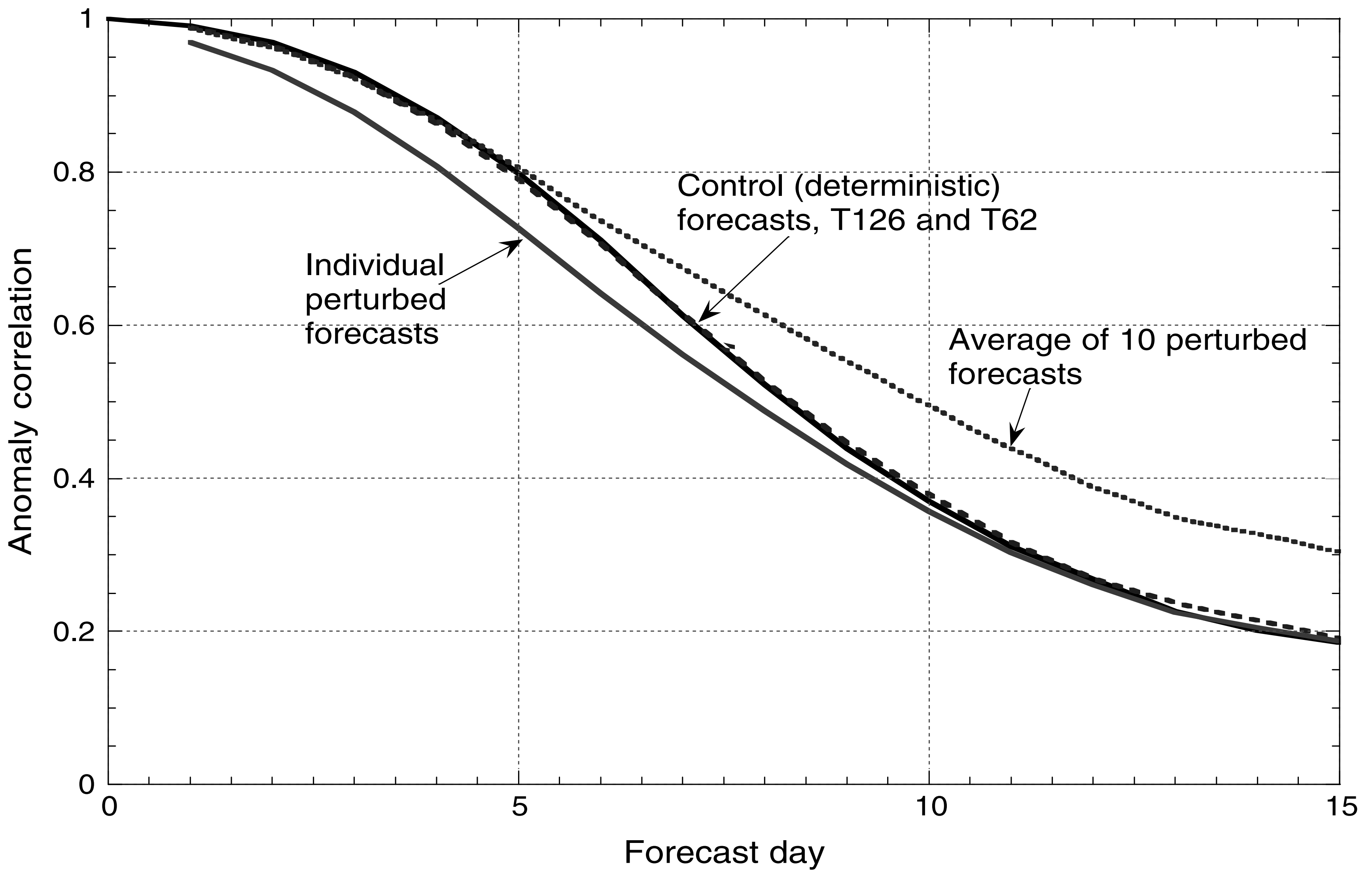
Short timescale (1 day)



Long timescale (7 days)

Kalnay, *Atmospheric Modeling, Data Assimilation and Predictability*

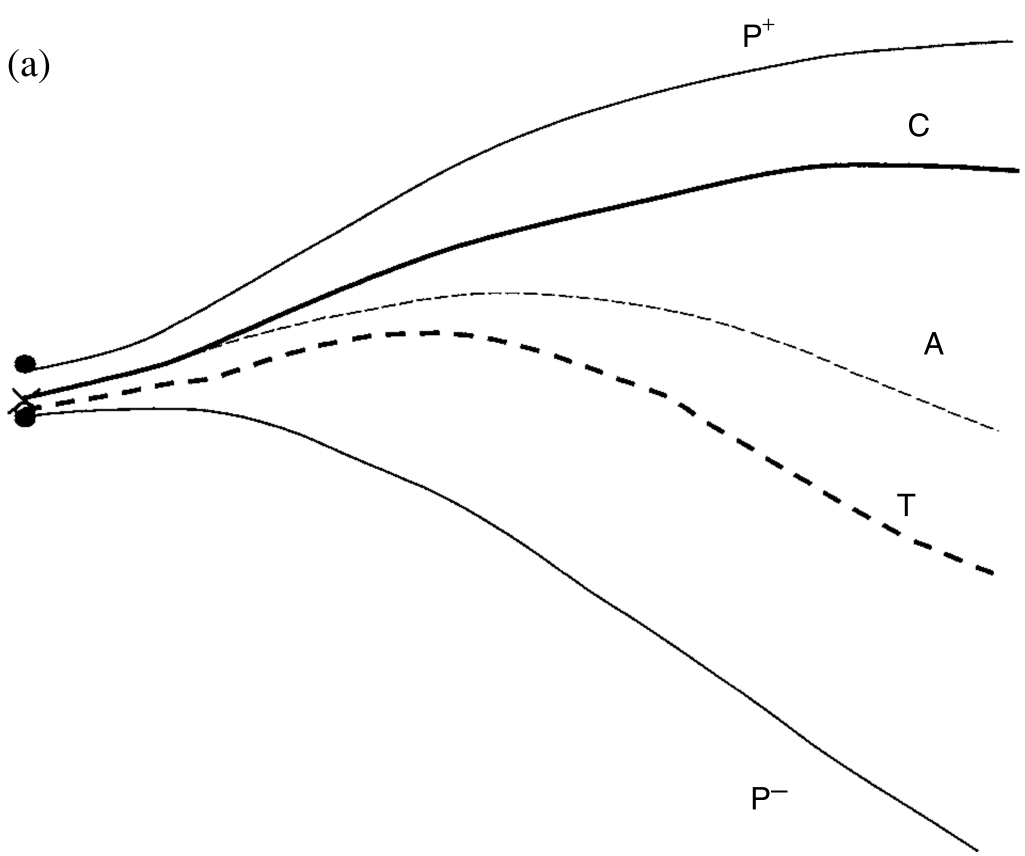
Performance of ensemble forecast



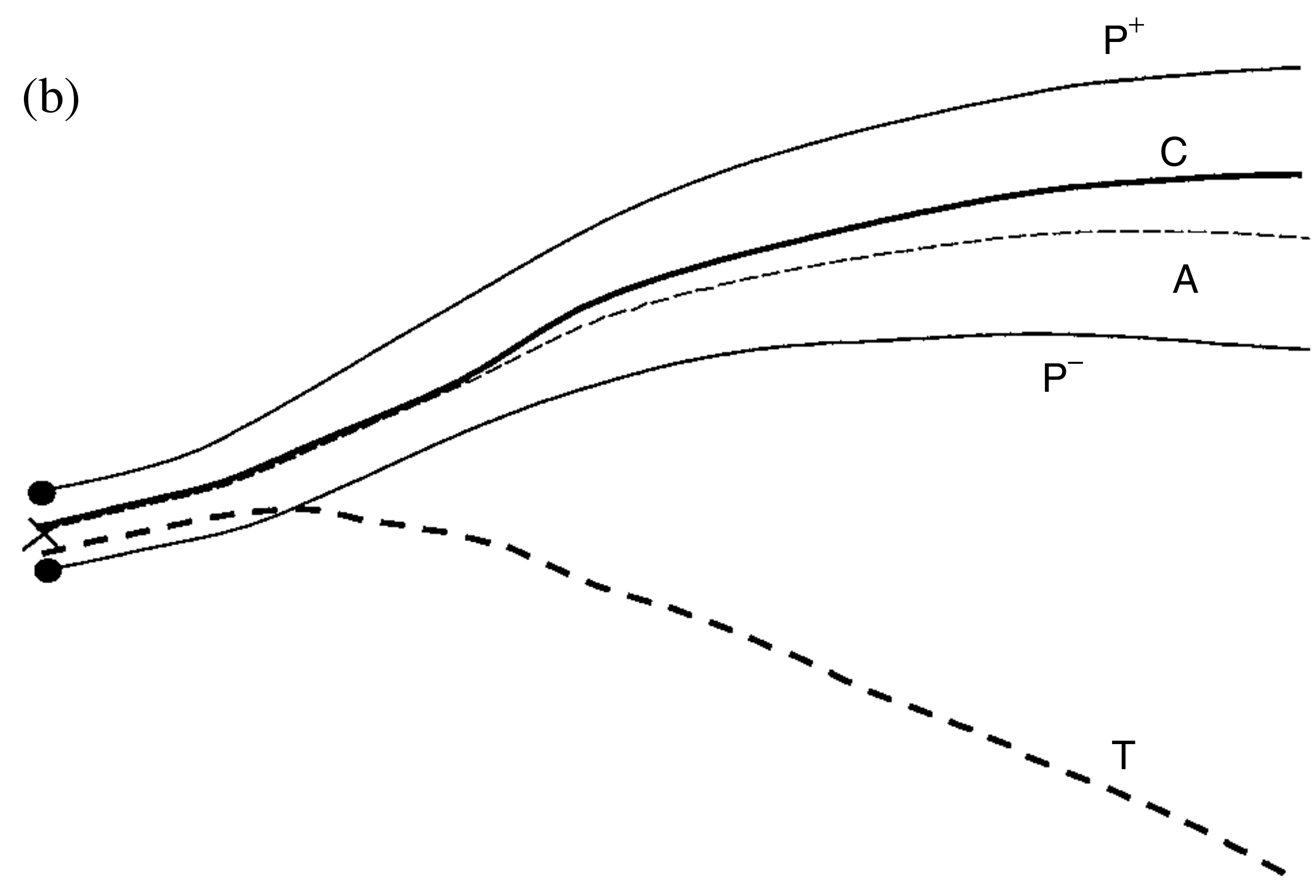
The ensemble-average forecast remains correlated with observations for longer than control or individual perturbed forecasts

Kalnay, *Atmospheric Modeling, Data Assimilation and Predictability*

Performance of ensemble forecast



Forecast error reduced by ensemble averaging



Forecast error dominated by systematic (model) error

Kalnay, *Atmospheric Modeling, Data Assimilation and Predictability*

III. Climate prediction
1. Predicting slow degrees of freedom

Timescales in the climate system

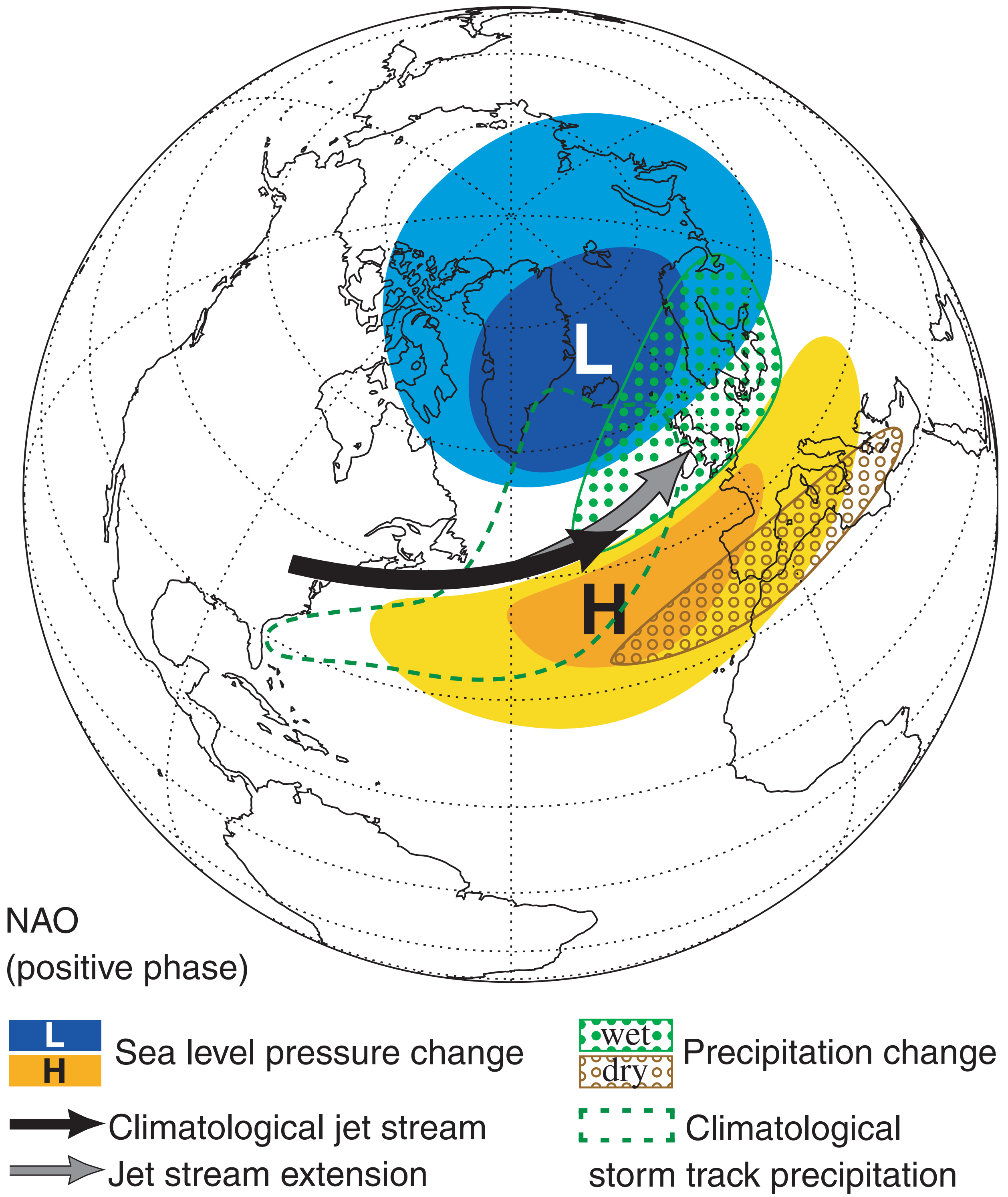
Timescale

Processes	days			years			thousands of years			millions of years	
	h/d	w	m	y	10 y	10^2y	10^3y	10^4y	10^5y	10^6y	10^9y
Weather	■	■									
Land surface	■	■	■								
Ocean mixed layer	■	■	■								
Sea ice		■	■	■							
Volcanos		■	■	■							
Vegetation	■	■	■	■	■	■	■	■	■	■	
Thermocline				■	■	■					
Mountain glaciers					■	■					
Deep ocean						■	■	■			
Ice sheets						■	■	■	■		
Orbital forcing								■	■		
Tectonics										■	■
Weathering									■	■	■
Solar "constant"				■	■	■	■	■	■	■	■

What are these slow processes and can they provide long range predictability?

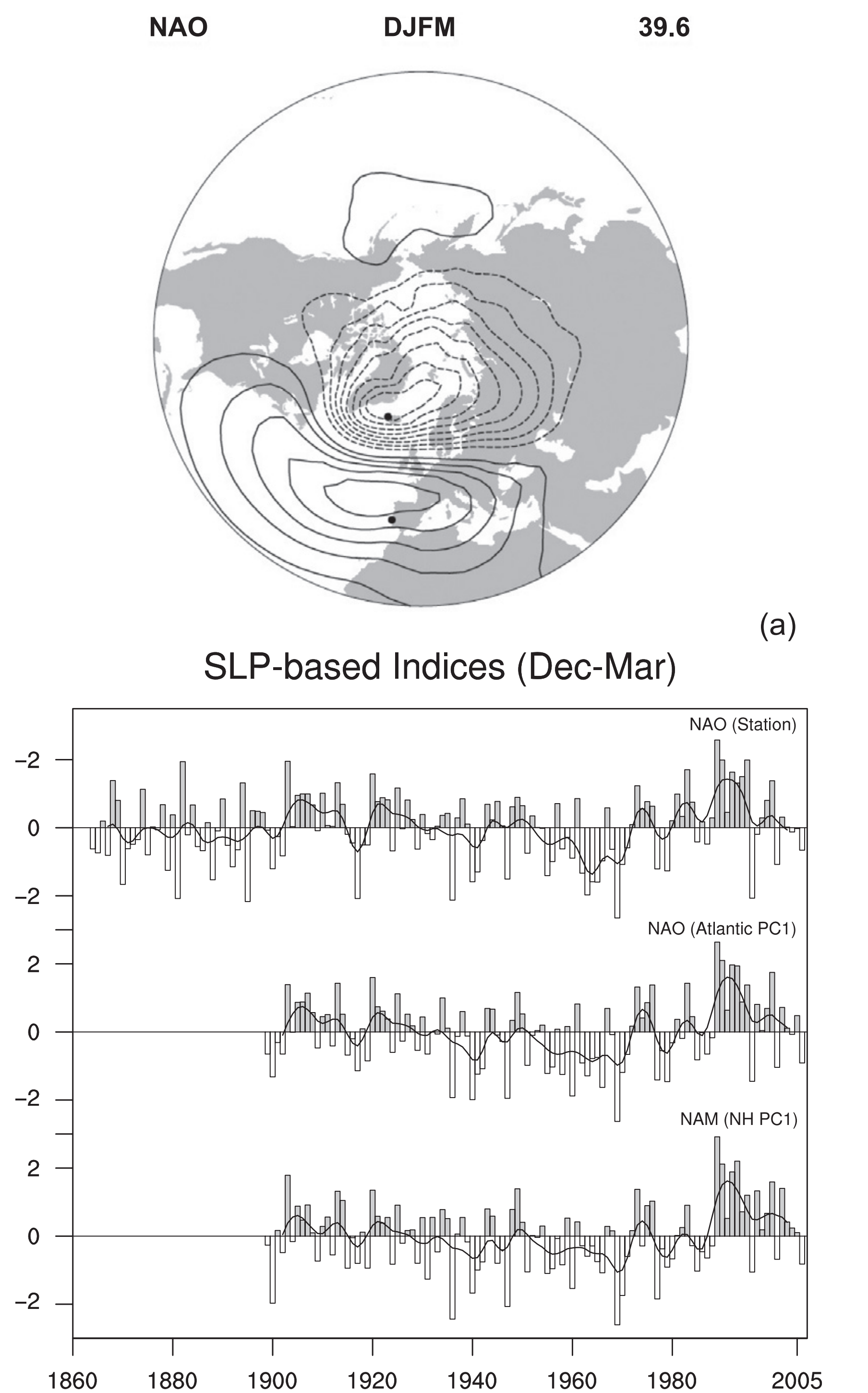
Marshall & Plumb, *Atmosphere, Ocean, and Climate Dynamics*

The North Atlantic Oscillation



Neelin, *Climate Change and Climate Modeling*

The North Atlantic Oscillation



Empirical Orthogonal Functions (or Principal Component Analysis)

For a random vector or field X, the covariance matrix is:

$$\boldsymbol{\Sigma} = \mathbb{E}[(\mathbf{X} - \boldsymbol{\mu})(\mathbf{X} - \boldsymbol{\mu})^T],$$

$$\boldsymbol{\mu} = \mathbb{E}[\mathbf{X}]$$

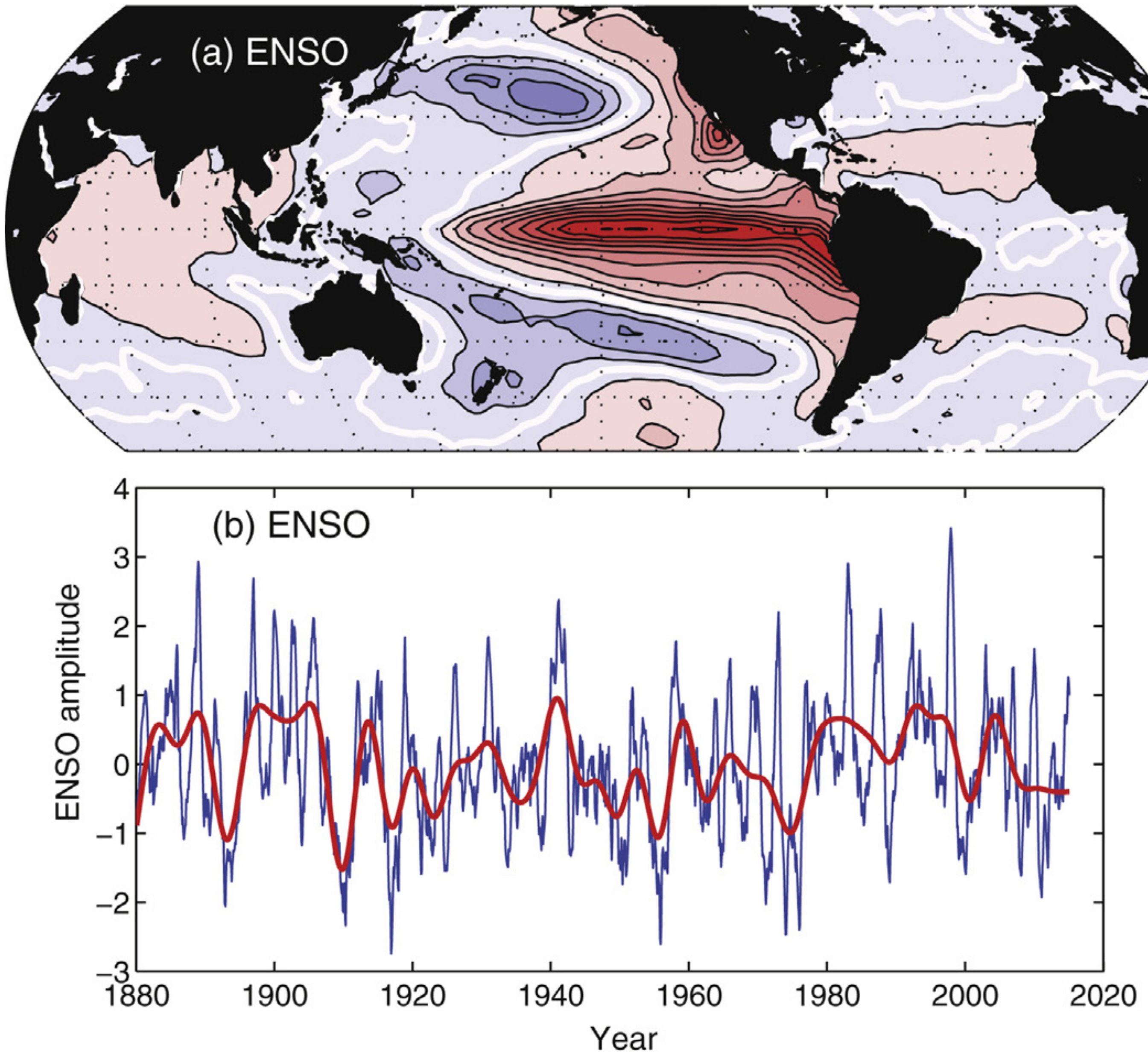
EOFs are the eigenvectors $\mathbf{e}_1, \ldots, \mathbf{e}_d$ of $\boldsymbol{\Sigma}$, ordered from larger to smaller eigenvalues $\lambda_1 \geq \ldots \geq \lambda_d$. Total variance is

$$\sigma^2 = \mathrm{Tr}\boldsymbol{\Sigma} = \sum_{i=1}^{d} \lambda_i$$

The field can be decomposed in this basis:

$\mathbf{X} = \sum_{i=1}^{d} \alpha_i \mathbf{e}_i$; each term explains $\frac{\lambda_i}{\sigma^2}$ % of the variance.

El Niño Southern Oscillation



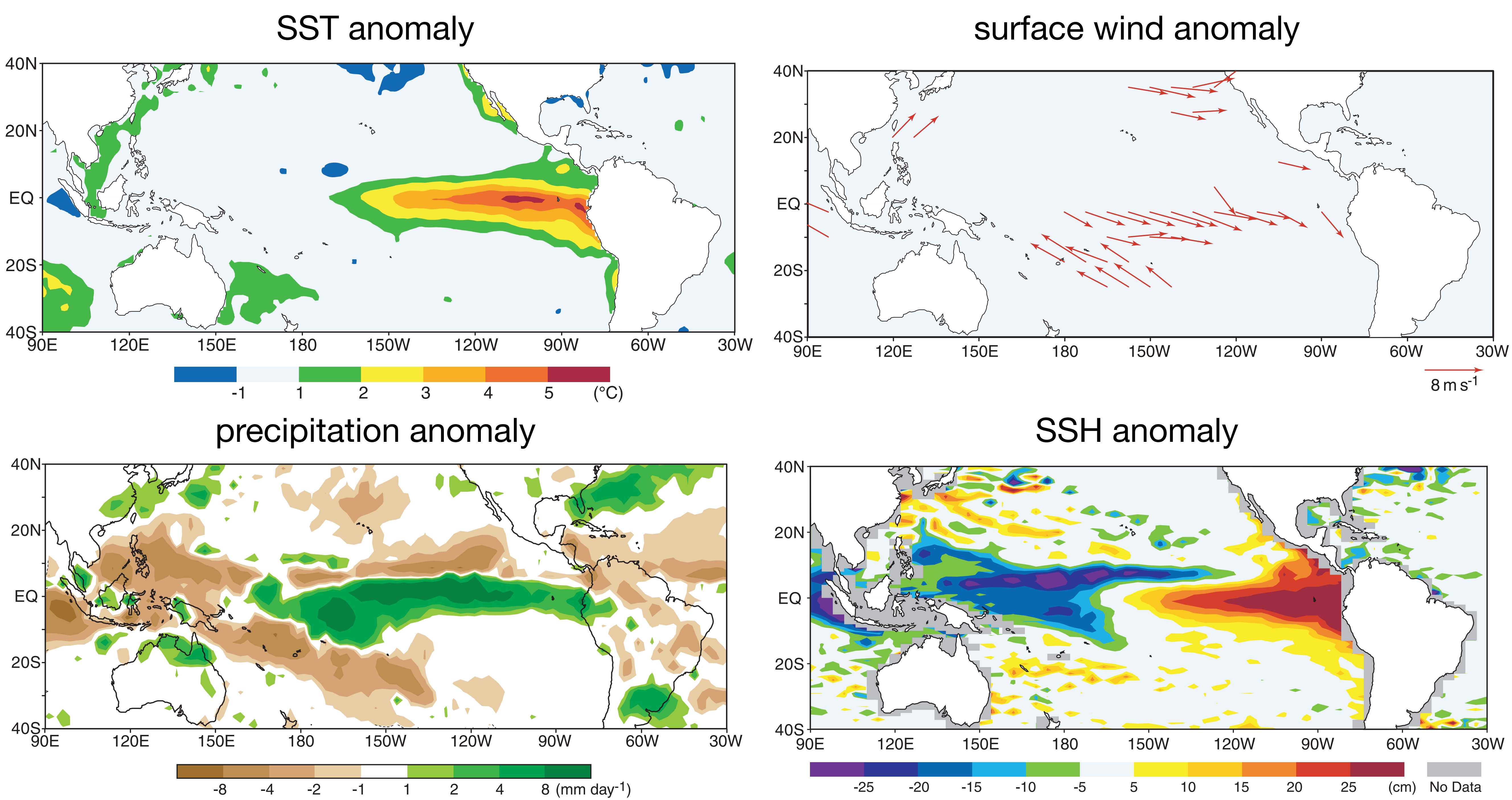
First EOF of global SST

Leading Principal Component

Leading mode of interannual climate variability

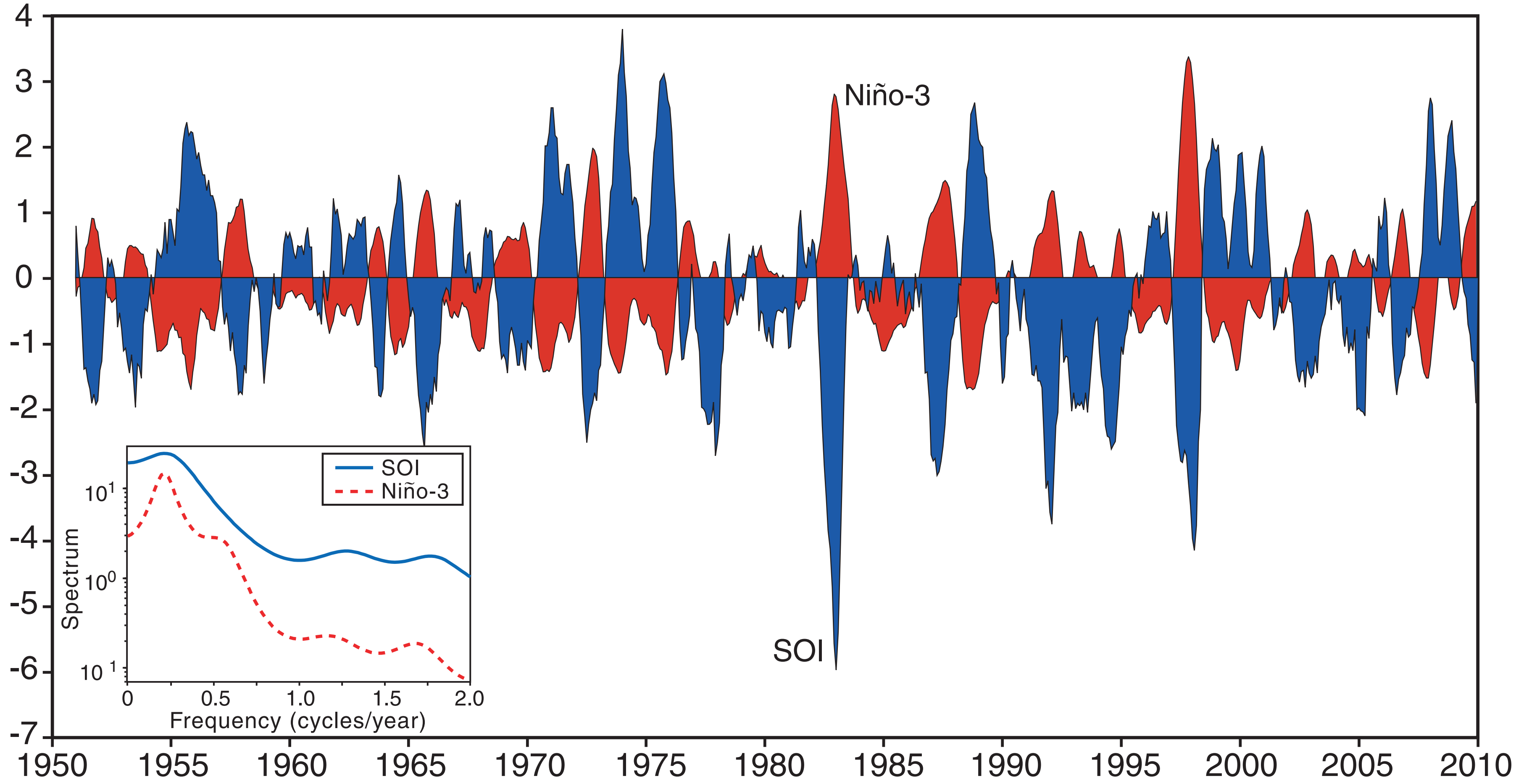
Hartmann, *Global Physical Climatology*

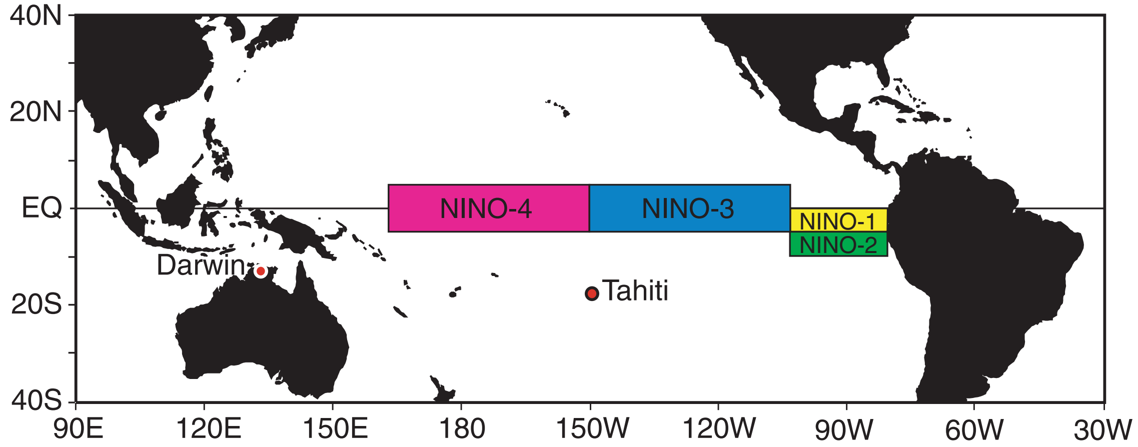
December 1997 El Niño event



Neelin, *Climate Change and Climate Modeling*

ENSO

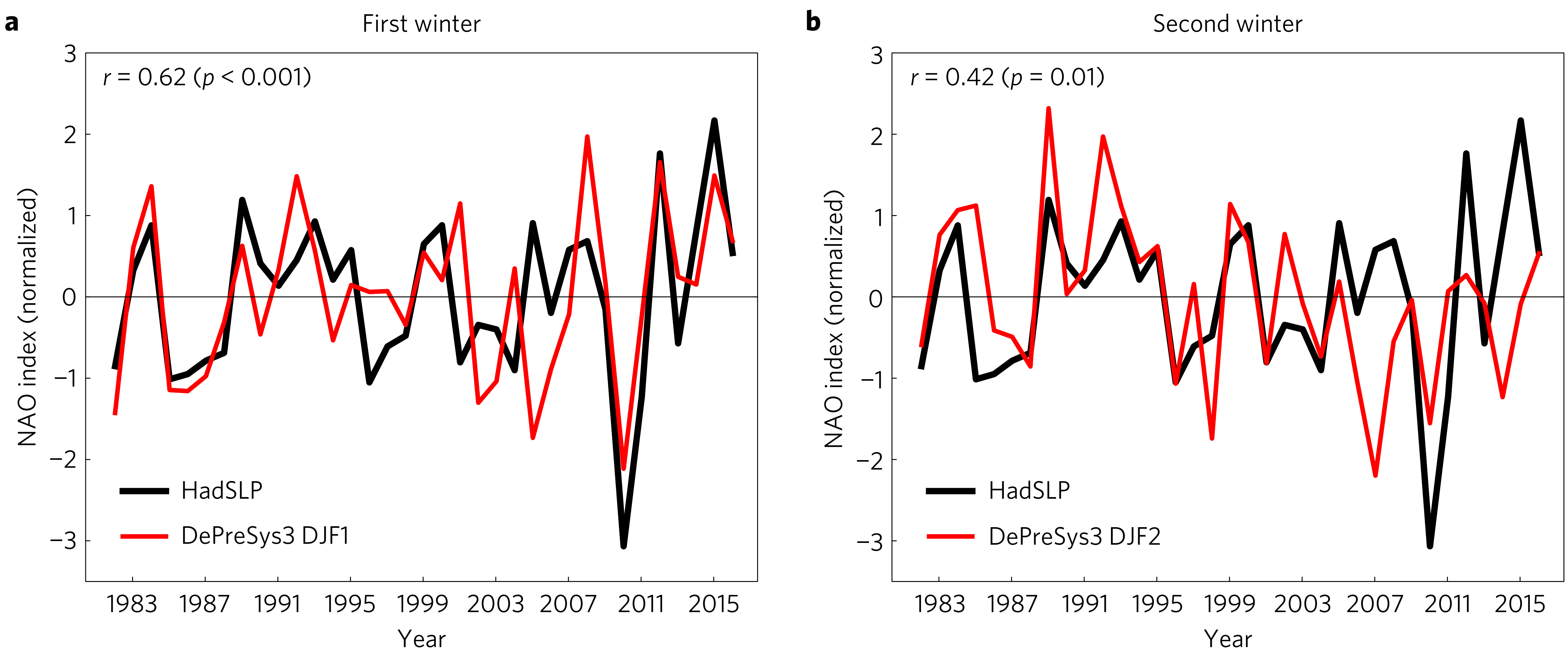




Neelin, *Climate Change and Climate Modeling*

Can we predict NAO?

NAO index: sea-level pressure difference between Iceland and the Azores

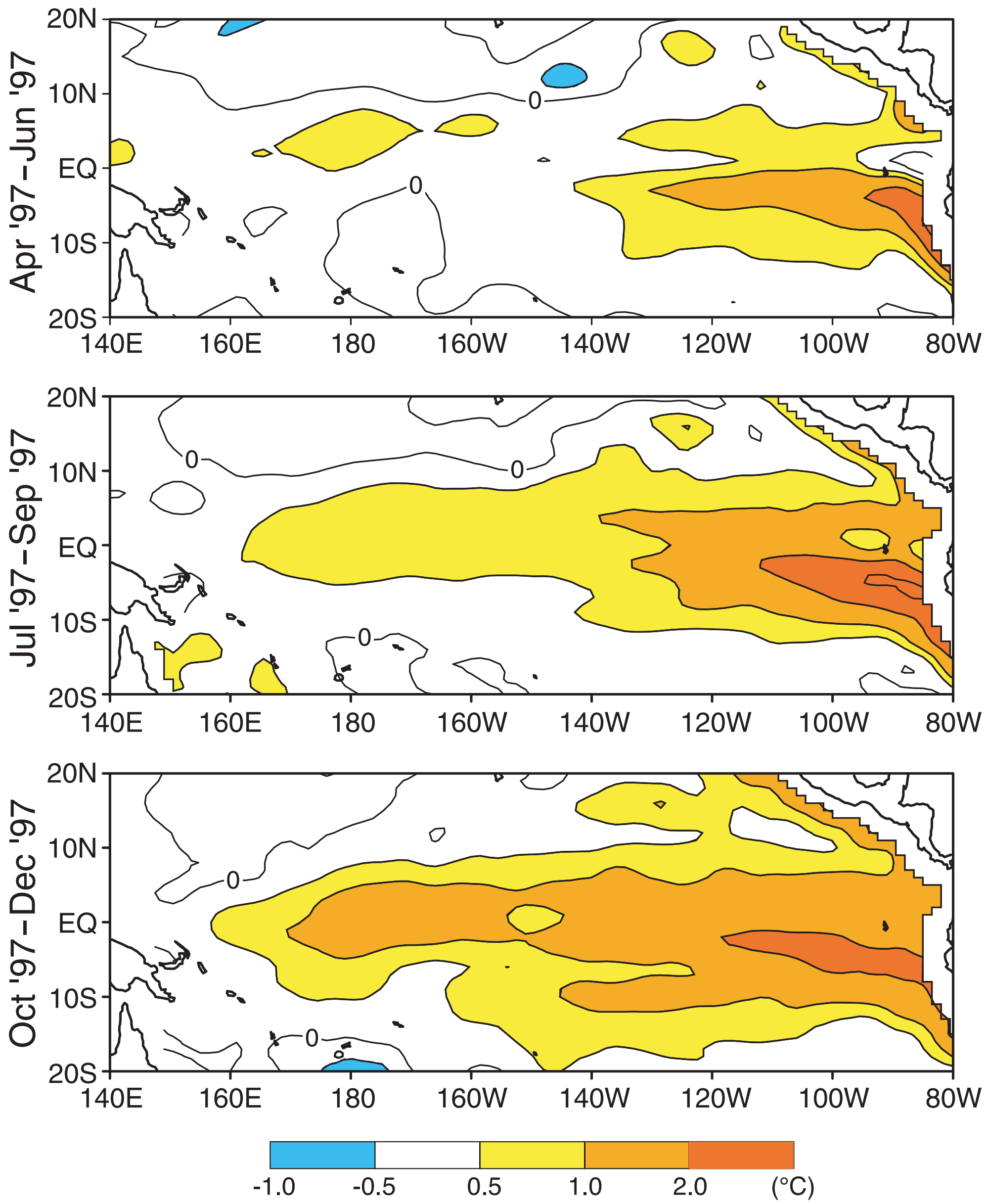


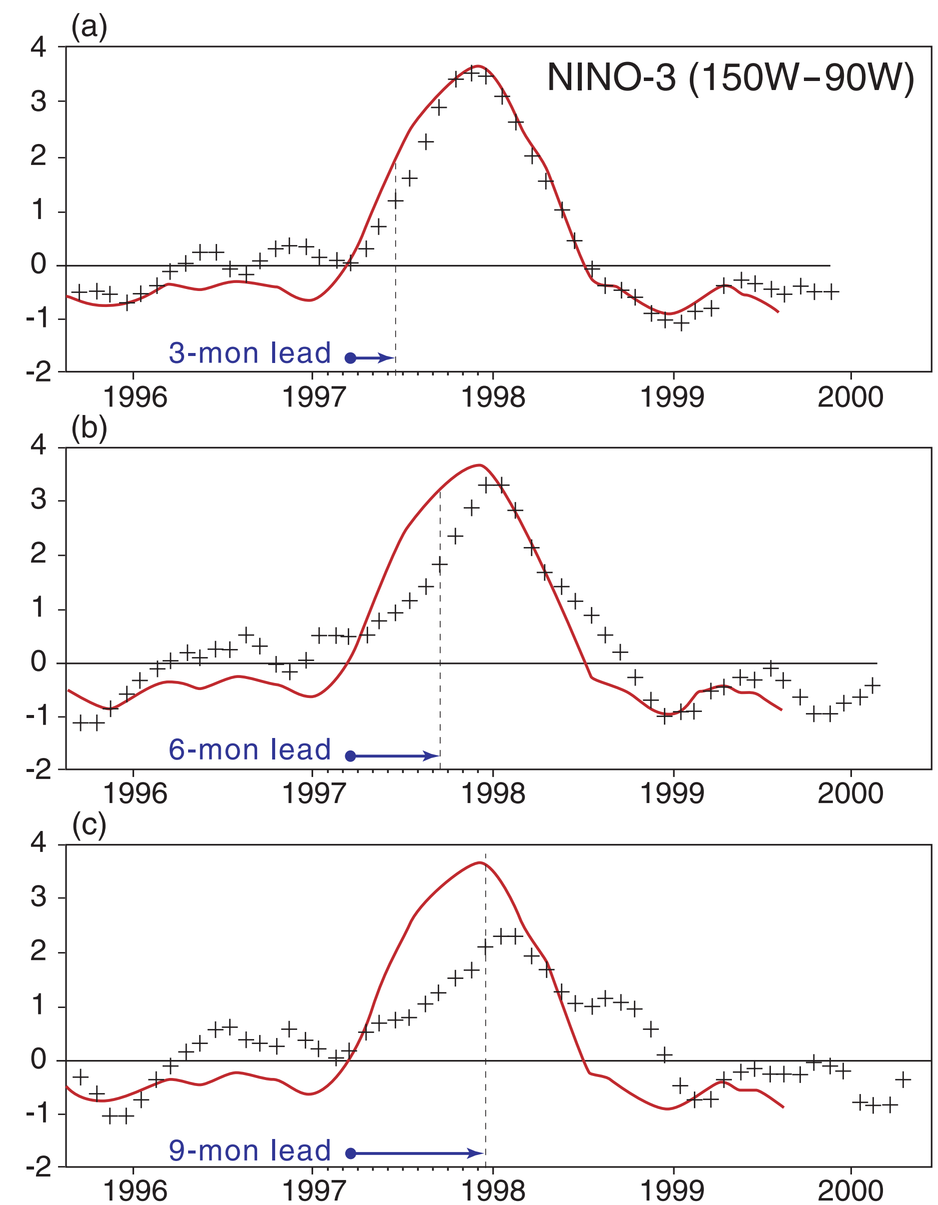
The ensemble-averaged prediction shows skill at predicting NAO a year or two in advance.

Dunstone et al. *Nature Geoscience* (2016)

Can we predict ENSO?

Forecast March 1997





There is some skill for predicting ENSO a few months in advance, but major obstacles remain ("spring predictability barrier"). This is a topic of active research.

Neelin, *Climate Change and Climate Modeling*

III. Climate prediction
2. Sensitive dependence on initial condition