Rotational dynamics of sedimenting anisotropic particles

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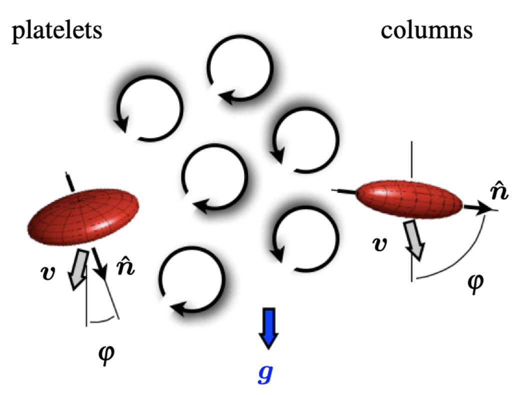
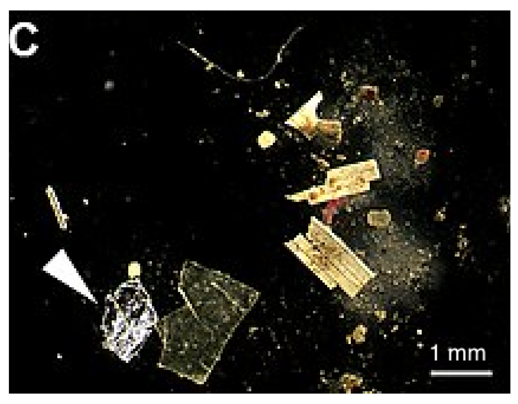
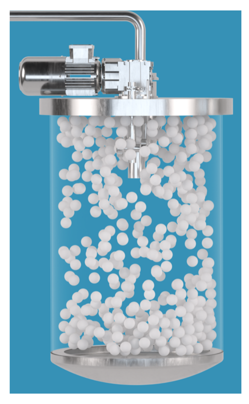
Why sedimentation of anisotropic particles ?

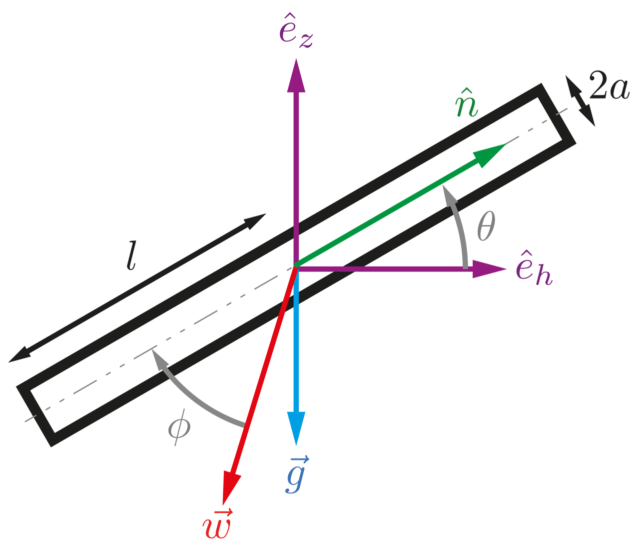
- **For Climate** :
  - physics of anisotropic ice crystals in clouds [3]
  - dynamics of micro-plastics and phytoplankton in Oceans [5]
- **For engineering and industries**:
  - catalysis (Fluidized Bed Reactor)







The governing equation for cylinders



Translational (4) and rotational (5) equations:

$$\vec{w} = \frac{d\vec{x}}{dt}; \quad m_p \frac{d\vec{w}}{dt} = m_p\vec{g} + \vec{f}_S + \vec{f}_I \tag{4}$$

$$J\ddot{\theta} = \underbrace{-C_S\dot{\theta}}_{\text{Stokes drag}} + \underbrace{\Gamma_I}_{\text{Inertial torque}} \tag{5}$$

Reynolds number $Re = \frac{|\vec{w}|l}{\nu}$; Aspect ratio $\beta = \frac{l}{a}$

Rotational dynamic of anistropic particles is an open question: what model for Γ_I?

- Dimensional analysis + symmetries consideration [2] [6]

$$\boxed{\Gamma_I = C_I(Re, \beta)|\vec{w}|^2 \sin(2\phi)} \tag{6}$$

- Asymptotic model [4] ($Re \to 0$, $\beta \to \infty$)

$$\boxed{C_I = \frac{5\pi\rho_f l^3}{3(\log\beta)^2}} \tag{7}$$

Our idea: using magnetic particles

Apply B field on magnetic cylinders (M)

$$(5) \to J\ddot{\theta} = -C_S\dot{\theta} + \Gamma_I + MB\cos\theta \tag{1}$$

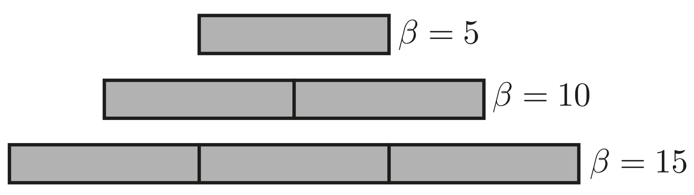
- Overdamped regime + steady flow:

$$\boxed{0 = \Gamma_I + MB\cos\theta} \tag{2}$$

$\Rightarrow$ Use B to probe Γ_I, hence $C_I(Re, \beta)$

Exploring parameter space (β, Re):

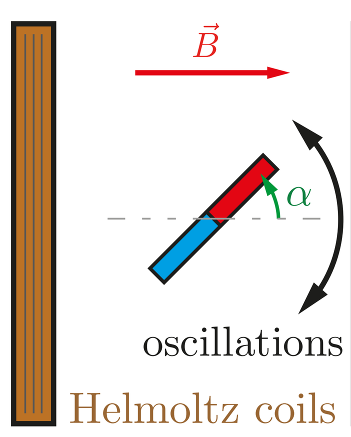
- Aspect ratios, $\beta = \{5, 10, 15\}$
- Magnet size, $Re \in [3, 45]$

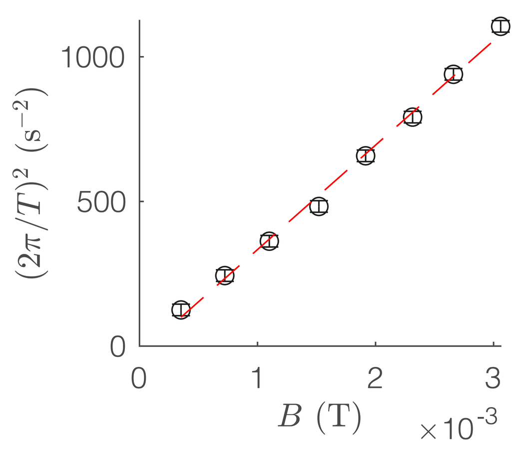


Magnetic cylinders

- Handmade magnetization : heating + remagnetization
- Magnetization measurement: oscillation method [1]

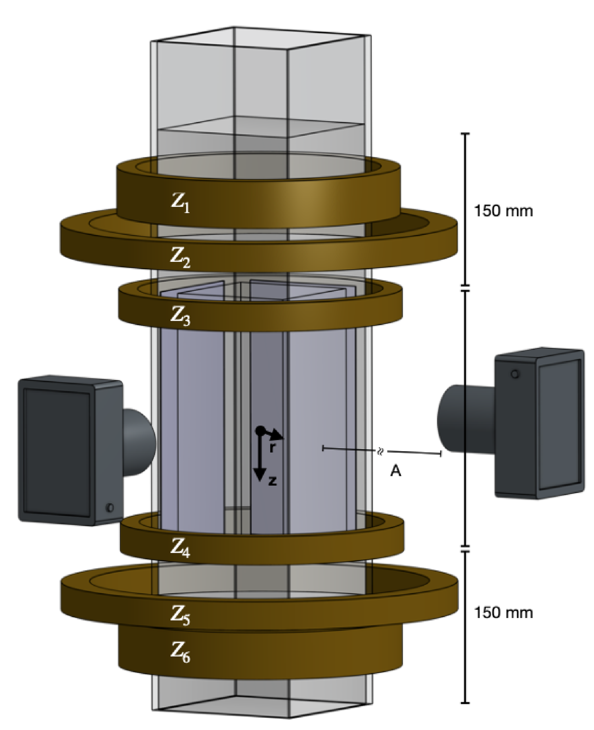
$$I\frac{d^2\alpha}{dt^2} = C\alpha + MB\sin\alpha \;\Rightarrow\; \frac{2\pi}{T} = \sqrt{\frac{C + MB}{I}} \tag{3}$$

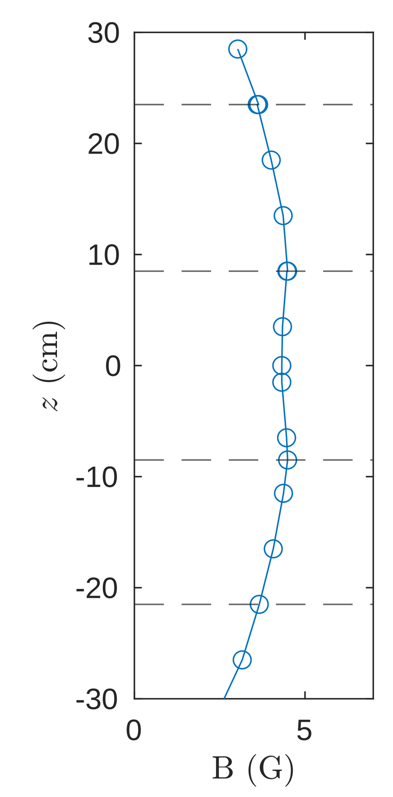


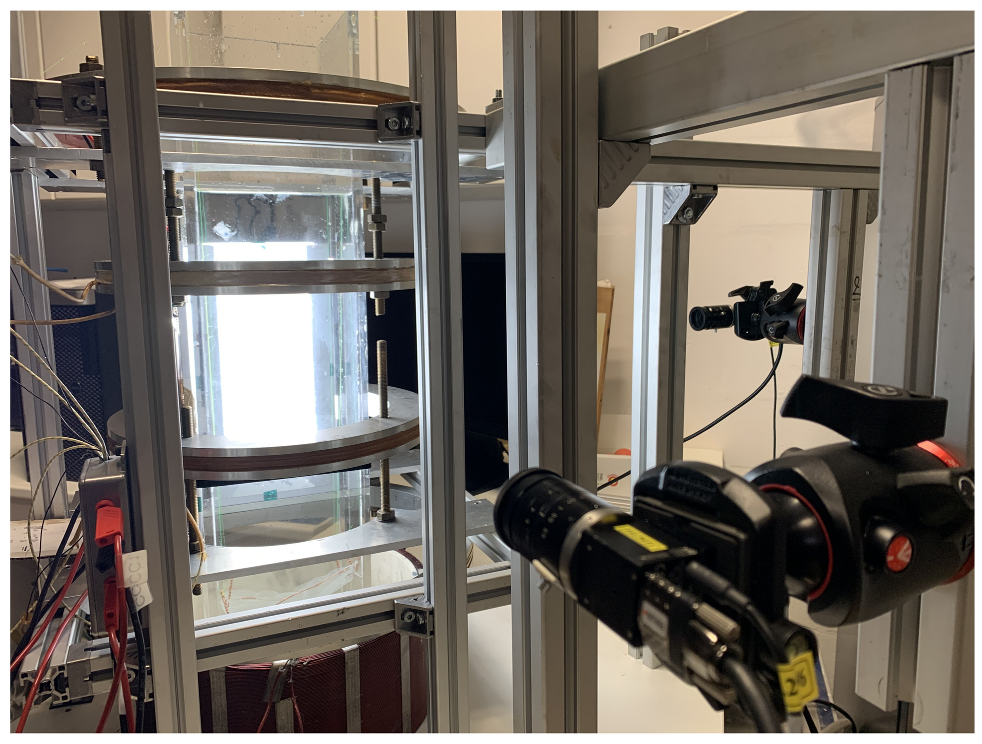


Experimental setup

- Magnetized cylinders settling in quiescent fluid
- Glycerol diluted solution ($\nu \approx 2, 5.10^{-4}$ Pa.s^{-1})
- Constant magnetic field: $\mathbf{B} = B\mathbf{e_z}$

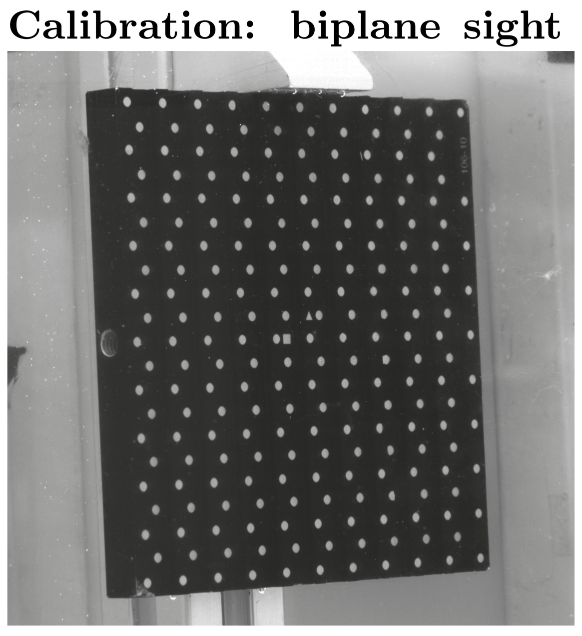


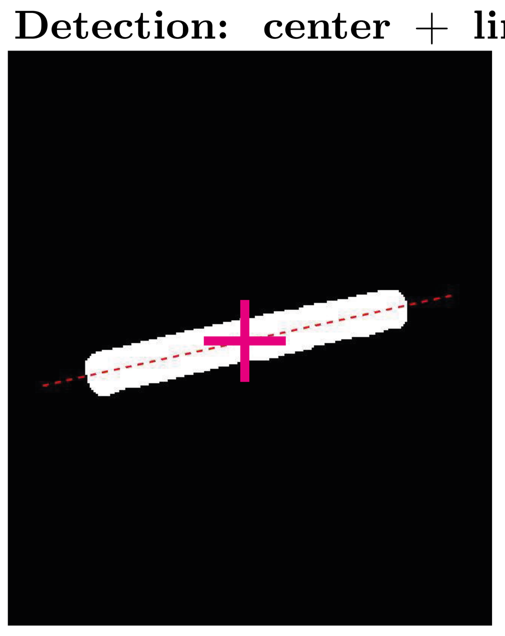




Particles tracking PTV

Calibration: biplane sight **Detection: center + line**



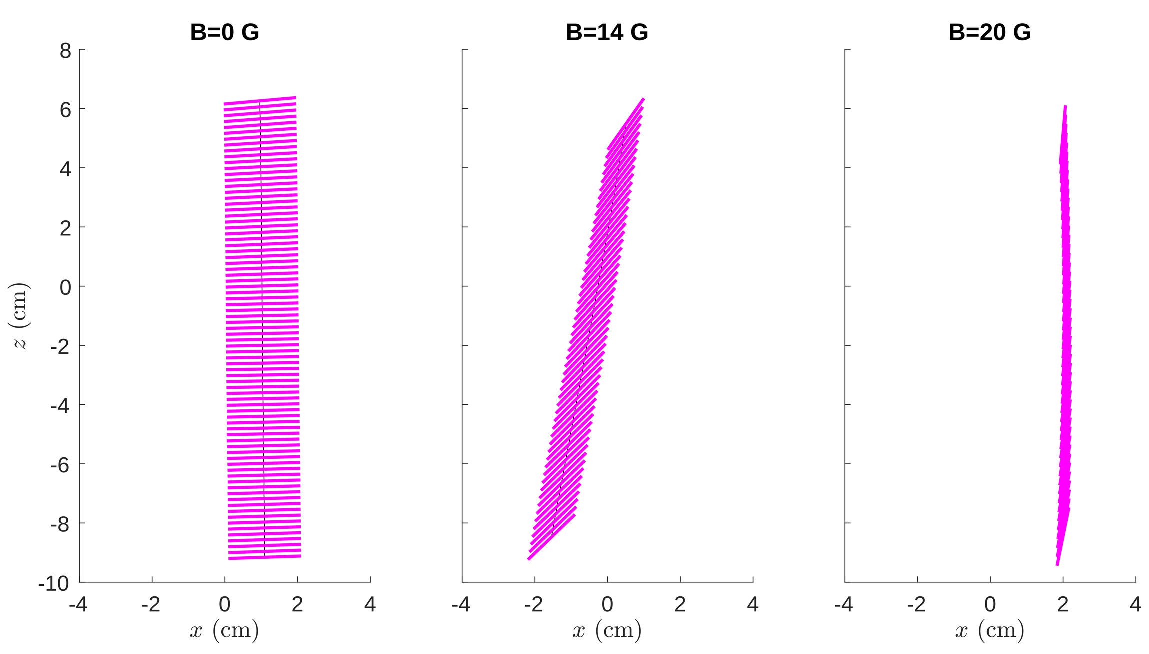


3D Tracking:

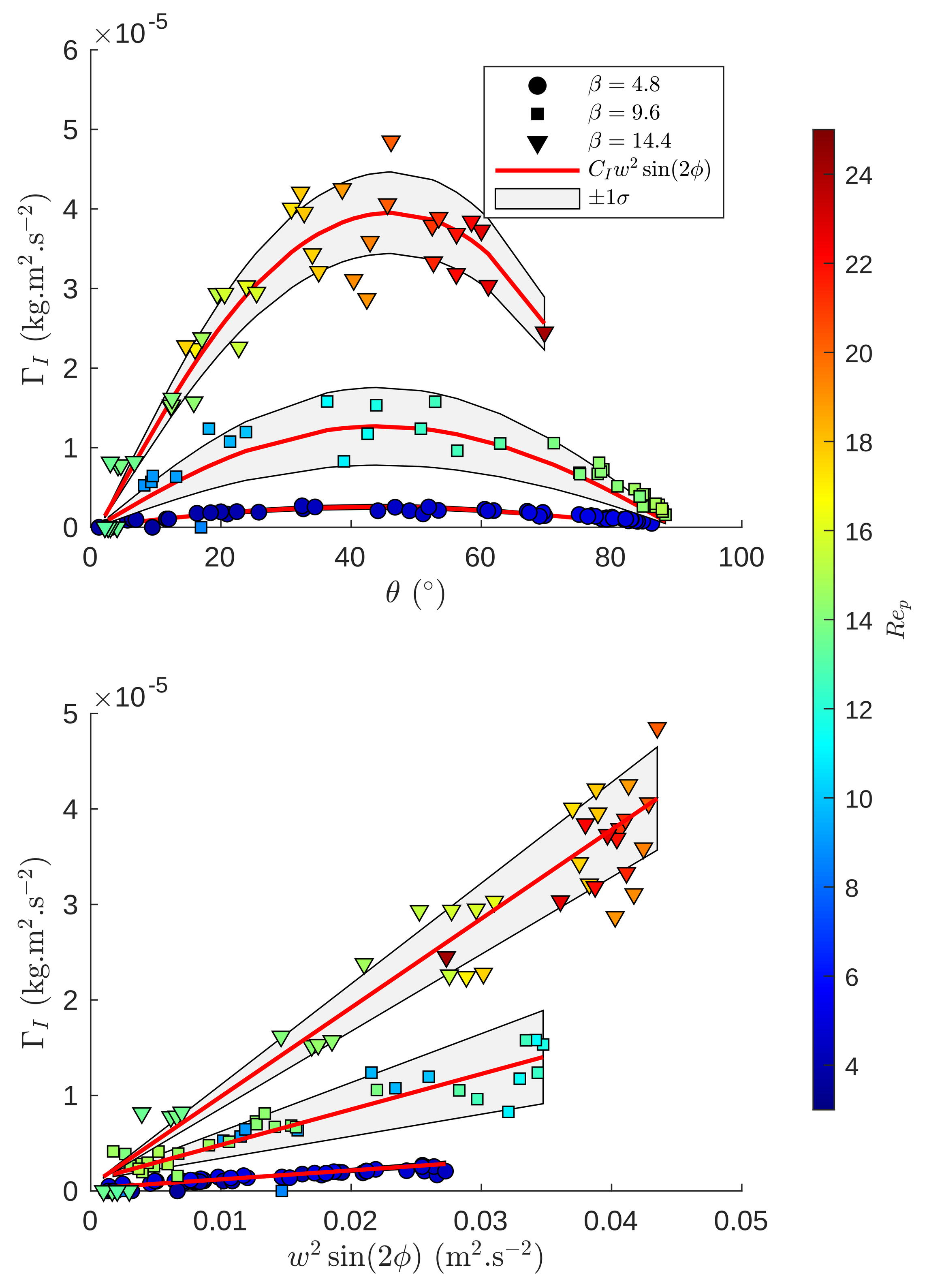
- For each camera : center + line $\Rightarrow$ plane
- 2 cameras $\Rightarrow$ plane intersection $\Rightarrow$ 3D position

$\Rightarrow$ Access to settling velocity & orientation

Examples of trajectories for different B:

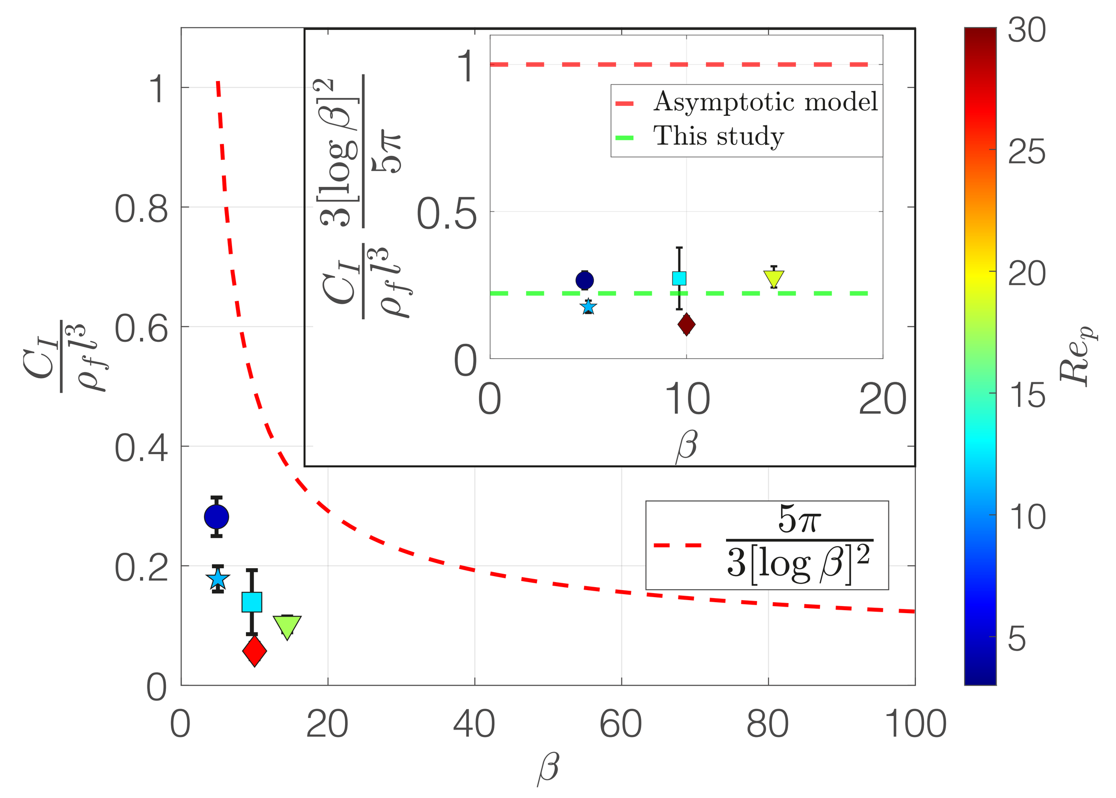


Inertial torque measurement, Γ_I



$\Rightarrow$ **Γ_I verify eq. (6)**

Shape factor, $C_I(\beta, Re)$



$\Rightarrow$ **C_I agrees with scaling of eq. (7) but with a prefactor**

Conclusion

- New setup and method to measure Γ_I and C_I
- Shape of Γ_I verify eq. (6)
- Qualitative **agreement with asymptotic prediction** eq. (7): **C_I independent of Re and scales as** $[\log\beta]^{-2}$, but $\frac{5\pi}{3}$ prefactor not verified

References

[1] B. Barman. Measuring the magnetization of a permanent magnet. *American Journal of Physics*, 87(4):275–278, April 2019.

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